Investigating the combined application of Common Random Numbers and Antithetic Variates

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OUTLINE

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INTRODUCTION (1)

- Variance Reduction Techniques: A procedure used to increase the precision (i.e. reduce standard deviation) of the estimates that can be obtained for a given number of iterations. The main ones are:
  - Common random numbers,
  - Antithetic variates,
  - Control variates and
  - Indirect method.

INTRODUCTION (2)

- The stand-alone and combined application of two variance reduction techniques which are based on manipulation of random number seeds on a Crossdocking distribution centre simulation model.
- Intuitively, combined application should perform better than stand-alone applications under the same circumstances. Can this statement be TRUE?.

METHODOLOGY

- Common Random Numbers: It entails dedicating a different stream of random numbers, different from the default set up in most simulation software, to each source of randomness in the simulation model.
- Antithetic Variates: It entails inducing a negative correlation between replications by using complimentary random numbers to drive two replications in a pair.

CASE STUDY(1)

- Crossdocking: A 'Just in time' Warehouse technique for receiving and shipping consignments without storage.
- Crossdocking distribution centres are typically characterised by random behaviour i.e. machine failure, which makes it suitably modelled by discrete event simulation.

CASE STUDY(2)

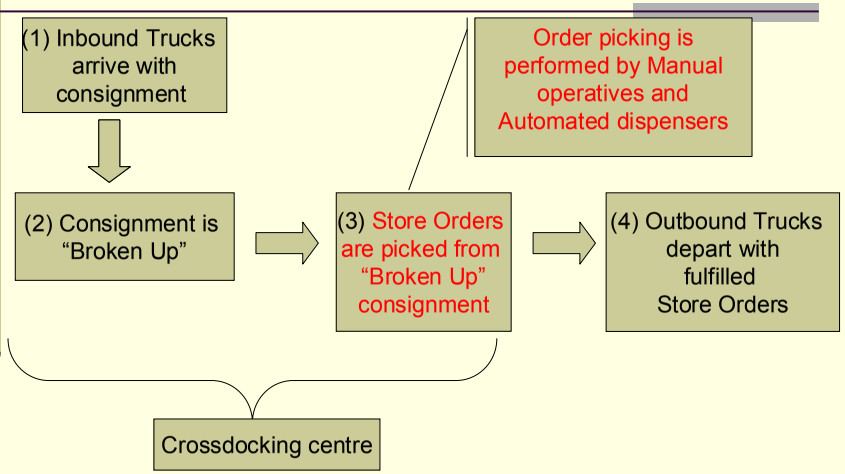

Figure 1 Typical crossdock flow of activity

CASE STUDY(3)

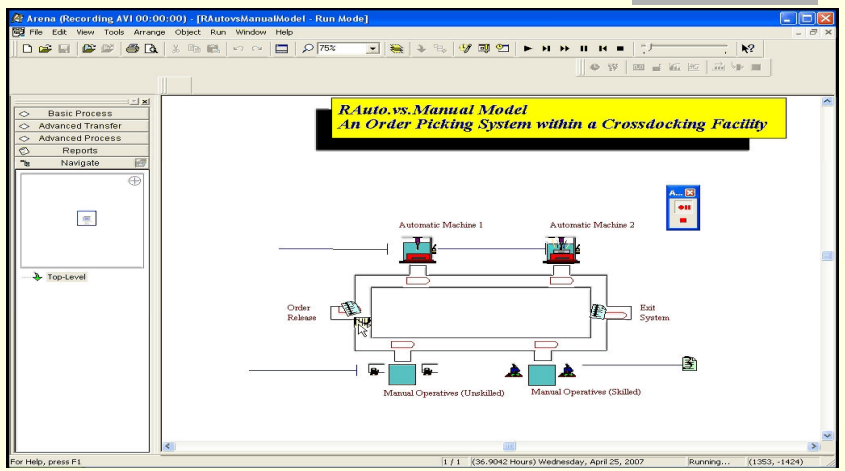

Figure 2 Simulation Model Implementation using Arena Software

EXPERIMENTS

- Null Hypothesis: Given random samples from independent discrete event simulation models, with normal distributed means:
  - Is Standard Deviation (Independent) < Standard Deviation (CRN)
  - Is Standard Deviation (Independent) < Standard Deviation (AV)
  - Is Standard Deviation (Independent) < Standard Deviation (Combined application)
  - Statistical test:
    - F test
  - Significant level (α) = 0.05
- Number of Simulation replications: 100, 500, 1000
- Performance measure: Total Resource Cost

RESULTS

	Independent	Common Random Numbers	Antithetic Variates	Combined Application	Number of Simulation Replications
Std. Dev.	1,413	1,304	987	977	100
P value		0.429	0.007	0.004	
Std. Dev.	1,386	1,296	995	950	500
P value		0.135	0.000	0.000	
Std. Dev	1,395	1,334	990	933	1000
P value		0.155	0.000	0.000	

CONCLUSIONS

For a test at 5% significance level:

- The P values (0.429), (0.135), (0.155) - CRN,

are more than a reasonable choice of (α) = 0.05.

- The P values (0.007), (0.000), (0.000) - AV, and
- The P values (0.004), (0.000), (0.000) - Combined application,

are less than a reasonable choice of (α) = 0.05;

Therefore,

1. We can fail to reject the null hypothesis and conclude that the standard deviation of Independent is less than the standard deviation of CRN, and
2. We can reject the null hypothesis and accept the alternative hypothesis that the standard deviation of AV is less than the standard deviation of Independent, and
3. We can reject the null hypothesis and accept the alternative hypothesis that the standard deviation of Combined application is less than the standard deviation of Independent, and
4. Antithetic Variates performs (i.e. smaller standard deviation) just as well as Combined application.

SUMMARY

The Combined application and Antithetic variates perform just as good as each other, followed by Common random numbers in reducing the variance of our Crossdocking distribution centre simulation model output performance measure. These results buttress the opinion of Kleijnen (1975), (1974), that Antithetic variates can be chosen, a priori, as a variance reduction technique for simulation, if simulation computational effort is an initial consideration.

QUESTIONS