Type-1 or Interval Type-2 Fuzzy Logic Systems - On the Relationship of the Amount of Uncertainty and FOU Size

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Outline

• Type-2 Fuzzy Sets and Systems
• Background
• Our Approach
• The FLSs Design and Evaluation Process
• Discussion & Results
• Conclusion
• Future Work
• Questions?
Type-2 Fuzzy Sets

- T1 FS: crisp degrees of membership
- T2 FS: fuzzy degrees of membership, a fuzzy-fuzzy set.

![Diagram of Type-2 Fuzzy Sets](image)
Type-2 Fuzzy Sets

- The FOU is the union of all primary memberships
- It is the region bounded by all of the primary memberships values i.e. the grey shaded region
- FOU is useful because:
  - Let us focuses on uncertainties .
  - Allows us to represent a type-2 FS graphically in 2 D instead of 3D.
Type-2 Fuzzy Systems

Crisp Inputs

Fuzzifier

Rule base

Defuzzifier

Crisp Output

T2 FSs

Inference Engine

Type-reduced T1 FS

Type-Reducer

T2 FSs

this is extra compared to type-1
Background

• Many researchers argue in favour of IT2 FLSs over T1 FLSs. Why?
  • Their potential to model and mitigate the effects of uncertainty.
  • Training data is corrupted by noise and the situation is too uncertain.
  • To handle the large amounts of uncertainties present in real world environments.
• Large amounts of uncertainty!
Background

• Mentioned in many of the papers considering T2 FSs.
• They expected to perform well in the face of “high levels of uncertainty”. But,
  • No systematic way of determining the potential advantage of employing T2 FSs over T1 FSs.
  • Did not quantifying what “high” means in general or in the case of their specific application.
Our Approach

• An application driven investigation into the relationship between the FOU size of FSs and the level of uncertainty.
• An application to Time Series Prediction (TSP)
• Design T1 FLSs for TSP.
• Then, generating different IT2 FLSs with gradually increasing FOU sizes over a number of steps.
Our Approach

- In parallel, we inject increasing levels of noise.
- This provide known and well defined/understood source of uncertainty.
- This approach shed light on the appropriate size of the FOUs for given levels of uncertainty/noise.
FLSs Design and Evaluation Process

• The design methodology
  • Step1: Generate training and testing data from the system under study
  • Step2: Design an initial T1 FLS.
  • Step3: Extend the T1 FLS into an IT2 FLS using the partially dependent design.
  • Step4: Performance Testing and Evaluation
FLSs Design and Evaluation Process

Step 1: Generate training and testing data from the system
- Mackey-Glass time series

Graphs showing data at different dB levels: 20 dB, 10 dB, 6 dB, 0 dB.
FLSs Design and Evaluation Process

Step 2: Design an initial T1 FLS

- 4-inputs 1-output

  - we apply the Wang-Mendel (WM) method in order to generate the rules from the given input-output pairs (training data).

  - The resulting rules are used for all FLSs used in our experiments
FLSs Design and Evaluation Process

Step 3: Extend the T1 FLS into an IT2 FLS

- The FOU size $c \in [0,1]$ is discretized to a set of $k$ values.
- Also, $k$ defines the number of noise/uncertainty levels that will be investigated.
- In our case $k = 11$
- $c = \{0, 0.1, 0.2, \ldots, 1.0\}$
- SNR = \{20, 18, 16, \ldots, 0\} dBs
FLSs Design and Evaluation Process

Examples:
IT2 membership functions used in inputs and output of IT2 FLSs with different FOU size

![Graph showing IT2 membership functions with c=0.30 and c=0.80]
FLSs Design and Evaluation Process

Step 4 : Performance Testing and Evaluation

- the performance testing using the pre-generated testing data at each of the k noise/uncertainty levels.
- At each noise level, the performance testing is repeated a number of times (in our case: 30 times).
- The performance of the design(s) is evaluated using the root mean-squared error (RMSE).
- The average of the RMSEs is then calculated of all of the iterations for each FLS at each noise/uncertainty level.
## Discussion and Results

### THE AVERAGE RMSE VALUES (FOR 30 TEST RUNS)

<table>
<thead>
<tr>
<th>IT2 FS Sample</th>
<th>0</th>
<th>0.10</th>
<th>0.20</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
<th>0.90</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter $c$ (FOU size)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.04148</td>
<td>0.04017</td>
<td><strong>0.03922</strong></td>
<td>0.03988</td>
<td>0.04208</td>
<td>0.04314</td>
<td>0.04357</td>
<td>0.04837</td>
<td>0.06365</td>
<td>0.08708</td>
<td>0.10913</td>
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<tr>
<td>18</td>
<td>0.04683</td>
<td>0.04656</td>
<td><strong>0.04519</strong></td>
<td>0.04520</td>
<td>0.04666</td>
<td>0.04732</td>
<td>0.04800</td>
<td>0.05425</td>
<td>0.06975</td>
<td>0.09207</td>
<td>0.11125</td>
</tr>
<tr>
<td>16</td>
<td>0.05415</td>
<td>0.05329</td>
<td>0.05168</td>
<td><strong>0.05129</strong></td>
<td>0.05239</td>
<td>0.05322</td>
<td>0.05409</td>
<td>0.05969</td>
<td>0.07382</td>
<td>0.09412</td>
<td>0.11222</td>
</tr>
<tr>
<td>14</td>
<td>0.06391</td>
<td>0.06271</td>
<td>0.06109</td>
<td><strong>0.06041</strong></td>
<td>0.06114</td>
<td>0.06185</td>
<td>0.06312</td>
<td>0.06844</td>
<td>0.08054</td>
<td>0.09887</td>
<td>0.11601</td>
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<tr>
<td>12</td>
<td>0.07673</td>
<td>0.07603</td>
<td>0.07425</td>
<td><strong>0.07302</strong></td>
<td>0.07303</td>
<td>0.07377</td>
<td>0.07551</td>
<td>0.08000</td>
<td>0.09075</td>
<td>0.10597</td>
<td>0.12076</td>
</tr>
<tr>
<td>10</td>
<td>0.09319</td>
<td>0.09292</td>
<td>0.09103</td>
<td>0.08954</td>
<td><strong>0.08934</strong></td>
<td>0.08977</td>
<td>0.09130</td>
<td>0.09505</td>
<td>0.10296</td>
<td>0.11588</td>
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<tr>
<td>8</td>
<td>0.11422</td>
<td>0.11434</td>
<td>0.11238</td>
<td>0.11085</td>
<td>0.10984</td>
<td><strong>0.10959</strong></td>
<td>0.10984</td>
<td>0.11132</td>
<td>0.11665</td>
<td>0.12748</td>
<td>0.13964</td>
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<tr>
<td>6</td>
<td>0.14788</td>
<td>0.14337</td>
<td>0.13869</td>
<td>0.13461</td>
<td>0.13305</td>
<td>0.13103</td>
<td>0.12981</td>
<td><strong>0.12905</strong></td>
<td>0.13065</td>
<td>0.13828</td>
<td>0.14848</td>
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<tr>
<td>4</td>
<td>0.22032</td>
<td>0.21841</td>
<td>0.19130</td>
<td>0.17781</td>
<td>0.16168</td>
<td>0.15487</td>
<td>0.15128</td>
<td>0.14904</td>
<td><strong>0.14808</strong></td>
<td>0.15265</td>
<td>0.15915</td>
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<tr>
<td>2</td>
<td>0.33157</td>
<td>0.31884</td>
<td>0.28397</td>
<td>0.25829</td>
<td>0.23738</td>
<td>0.21375</td>
<td>0.19765</td>
<td>0.18431</td>
<td>0.17378</td>
<td><strong>0.17016</strong></td>
<td>0.17448</td>
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<tr>
<td>0</td>
<td>0.49397</td>
<td>0.44125</td>
<td>0.40718</td>
<td>0.37677</td>
<td>0.34288</td>
<td>0.31175</td>
<td>0.28386</td>
<td>0.25518</td>
<td>0.23685</td>
<td>0.22305</td>
<td><strong>0.21278</strong></td>
</tr>
</tbody>
</table>

**NF**

<table>
<thead>
<tr>
<th>SNR (dBs)</th>
<th>0</th>
<th>0.10</th>
<th>0.20</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
<th>0.90</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 Test Runs</td>
<td>0.03032</td>
<td>0.02973</td>
<td><strong>0.02935</strong></td>
<td>0.03101</td>
<td>0.03403</td>
<td>0.03552</td>
<td>0.03514</td>
<td>0.03971</td>
<td>0.05634</td>
<td>0.08190</td>
<td>0.10808</td>
</tr>
</tbody>
</table>

The table above presents the average RMSE values for different SNR (dBs) levels and parameter $c$ (FOU size) values for IT2 FS Sample.
Discussion and Results

An average RMSE values group of 11 IT2 FLSs over 11 noise levels
Discussion and Results

\[
\begin{align*}
&c=0.30 \\
&c=0.50 \\
&c=0.80
\end{align*}
\]
Conclusion

• We have found a direct relationship between the FOU sizes of the FSs and the noise levels.
• As the noise level increases, the FOU that gives the minimum RMSE value increases as well.
• While the performance analysis of the systems shows an expected degradation due to a large/excessive injection of noise to the data; with expanding the FOU size, the performance improves for cases with higher noise level.
• From this, a strong relationship between the FLSs performance and the FOU size is obvious.
• It has been observed that the IT2 FLSs outperform their counterpart T1 FLS in all cases presented in the experiments even in the case of noise free data.
Future Work

• A focus on further analyzing the relationship of increasing SNR levels and FOU size in regards to performance, in particular by considering different applications.
• Performing a more detailed statistical analysis of the results.
• More effective uncertainty identification and capturing methods for designing IT2 FLSs will be considered.
References

Questions / Comments

Thank You !