Type-1 OWA Operator based Non-Stationary Fuzzy Decision Support Systems for Breast Cancer Treatments

Shang-Ming Zhou, Jonathan M. Garibaldi, Francisco Chiclana, Robert I. John and Xiao-Ying Wang

***Abstract*— In this paper, a novel type-1 OWA based non-stationary fuzzy system is proposed, in which the type-1 OWA operator is used in the fuzzy inference process to aggregate the non-stationary fuzzy outputs. The advantage of non-stationary fuzzy sets lies in their ability to model expert's variations in automated decision support systems. The proposed scheme offers an opportunity to combine different uncertain objects with uncertain weights into an overall decision in the fuzzy inference process. The agreement achieved between the proposed fuzzy system and clinical expert decision in selecting optimal treatment plans is used to evaluate the performance of the method. The experimental results on post-operative breast cancer treatments have demonstrated that the proposed fuzzy system can effectively diagnose breast cancer treatment in decision supports.**

I. INTRODUCTION

Breast cancer is the most common cause of cancer death for women worldwide [1] [2]. The outcomes for patients depend critically on the timely diagnosis and quality of treatment given. It is widely accepted that developing a treatment plan for a breast cancer patient is a very complex process, many factors from patient and medical diagnosis need to be considered, like age, lymph node stage, tumour size, tumour grade, patient's preferences, and many more besides. On the other hand, medical diagnosis is innately uncertain. Patients cannot describe exactly what has happened to them or how they feel, doctors cannot tell exactly what they observe, laboratories report results with some degree of measurement errors, etc. All these factors make it difficult, even impossible in some cases, to decide exactly what should be done in every conceivable set of circumstances. So soft decision support system for breast cancer treatments has emerged to assist, not replace, the doctors in the process of medical diagnosis to select optimal treatment for breast cancer [3]–[5], in which fuzzy rules were used to characterise doctor's diagnosis knowledge under uncertain environments.

However, it has often been demonstrated, particularly in breast cancer decision making, that groups of experts exhibit both intra-expert variability and inter-expert variability [6], [7]. *Intra-expert variability* is exhibited when an individual expert's decisions, given the same problem (i.e. the same data), change over time. This variability is due to a number of factors. In some cases, it may be attributed to the fact that the expert has acquired new knowledge and become more experienced (i.e. the expert has learned). *Inter-expert variability* is exhibited when a group or panel of experts differ in their decisions in a particular situation (when faced with the same data). Recently, non-stationary fuzzy sets have been proposed to model this sort of variation [7], in which variability is introduced into the membership functions of a fuzzy set through the use of random alterations to the parameters of generating function(s). In this way, the membership function of a non-stationary fuzzy set may alter over time. As a result, given a problem, a non-stationary fuzzy system may generate different output fuzzy sets in different runs, which is different from the standard fuzzy system. So a natural problem arising in a non-stationary fuzzy system is: how to aggregate these different output fuzzy sets into an overall one so that the final result can take into account the effects of variability in non-stationary fuzzy sets?

Recently, Zhou et al suggested a type-1 ordered weighted averaging (OWA) operator to aggregate fuzzy sets with fuzzy weights [8], which can be used to aggregate the linguistic opinions or preferences in human decision making with linguistic weights. The purpose of this paper is to propose a novel type-1 OWA based non-stationary fuzzy system, in which the type-1 OWA operator is used in fuzzy inference process to aggregate the non-stationary fuzzy outputs. The agreement between the proposed fuzzy system and clinical expert decision will be used to evaluate the performance of the method. The experimental results on post-operative breast cancer treatments have demonstrated that the proposed fuzzy system can effectively diagnose the breast cancer cases in decision making.

The organisation of this paper is as follows. Section 2 describes the breast cancer treatment data collected from hospital. The non-stationary fuzzy sets and type-1 OWA operators are briefly reviewed in Section 3, and Section 4 proposes a type-1 OWA based non- stationary fuzzy system. Section 5 evaluates the performance of the proposed scheme with post-operative breast cancer treatment data. Then Section 6 concludes the paper.

II. CLINICAL DATA OF BREAST CANCER ADJUVANT TREATMENTS

The clinical data about breast cancer treatment used in this paper was collected from Nottingham Breast Institute at Nottingham City Hospital in the UK. This is a post-operative dataset recording the historical breast cancer adjuvant treatments for the patients who had all undergone certain form

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of breast cancer operation (e.g. wide local excision, axillary node clearance or sample). The objective of adjuvant therapy is to reduce the chance of cancer reoccurrence. This data set is comprised of some attributes examined on each patient's post-operative visit and adjuvant therapy treatment decision.

- Patients' identification, date of birth, visit and diagnosis date.
- Invasive carcinoma size, grade, type and margins.
- Lymph node stage, the number of positive lymph node (LN) found from samples, number of positive apical nodes from samples.
- Nottingham prognostic index (NPI) value.
- Estrogen receptor (ER) test result
- Progesterone receptor (PR) test result
- Ductal carcinoma in situ (DCIS) size, grade, type and margins.
- Vascular invasion (VI) test result
- Whole tumour size

The clinical procedure employed for recording the data can be summarised by the following steps: i) the attribute information and additional comments related to each patient's treatment are recorded on a form, ii) the forms are discussed during the multi-disciplinary meeting and a further course of action is agreed, iii) after the meeting, the forms are collected and sent to a data analyst for entry into a computer database.

In the multi-disciplinary meeting, the adjuvant treatment plans regarding the recommended course of follow up treatments are normally provided by a multi-disciplinary team from surgeons, pathologists and oncologists for the patients. The adjuvant therapy plans may include *hormone therapy*, *radiotherapy*, *chemotherapy*, *biological therapy*, *further operation* and *follow up* or any combination of the above. But in this paper we only focus on the treatment decision: whether *chemotherapy* is necessary given the states of a patient after operation?

In this paper, 1310 cancer cases having full clinical records are used. It is worth noting that several challenges arise whilst using the dataset for this study due to the clinical procedures for recording the information. For example, the data consists of the input from a number of different data analysts over a period of twenty-five years and, because there is no standardised format for data entry, inconsistencies in data formats can often occur. The processing of the data is further complicated by the fact that different treatment decisions may be identified through different notations or spellings. This affects the clarity of the data and complicates automatic classification. Also there are some missing values for some attributes due to medical practice and the multi-dimensional nature of the data.

III. Review of Some Concepts

A. Non-stationary fuzzy sets

Different from standard fuzzy sets, non-stationary fuzzy sets are new kinds of fuzzy sets whose membership functions altering over time. Formally, a non-stationary fuzzy set [7] is defined as follows:

Definition 1: A non-deterministic fuzzy set, denoted $\dot{A}$, is characterised by its membership function, $\mu_{\dot{A}}(x,t)$, where $x \in X$and $\mu_{\dot{A}}(x,t) \in [0,\ 1]$, and t is a free variable indicating the *time* at which the fuzzy set is instantiated, i.e.,

$$\dot{A} = \int_{x \in X} \mu_{\dot{A}}(x,t)/x \quad \mu_{\dot{A}}(x,t) \in [0,\ 1] \tag{1}$$

At any given moment of time, i.e., in any specific instantiation, $\dot{A}$ will instantiate a standard type-1 fuzzy set. For example, in the context of Gaussian membership functions, there exist three main alternative kinds of non-stationarity: Variation in centre

$$\mu_{\dot{A}}(x,t) = e^{\frac{-(x-c(t))^2}{\sigma^2}} \tag{2}$$

Variation in width

$$\mu_{\dot{A}}(x,t) = e^{\frac{-(x-c)^2}{\sigma(t)^2}} \tag{3}$$

Noise variation

$$\mu_{\dot{A}}(x,t) = e^{\frac{-(x-c)^2}{\sigma^2}} \pm \varepsilon(t) \tag{4}$$

where the $c(t)$, $\sigma(t)$, and $\varepsilon(t)$ can be generated respectively by the following equations:

$$c(t) = c + \kappa f(t) \tag{5}$$

$$\sigma(t) = \sigma + \kappa f(t) \tag{6}$$

$$\varepsilon(t) = \kappa f(t) \tag{7}$$

in which c and σ are the centre and width of the initial type-1 fuzzy set respectively, κ is a constant value, and $f(t)$ is a perturbation function that generates small changes in the base membership function. In theory, $f(t)$ could be a true random function, i.e., the membership function parameter could be a true random variable: hence, the terminology of *non-stationary* fuzzy sets occurs.

B. Type-1 OWA operators

The departure point for defining such an uncertain OWA operator [8] is to aggregate the linguistic variables (modelled as fuzzy sets) used to express human opinions or preferences in soft decision making.

Definition 2: Given n linguistic weights $\{W_i\}_{i=1}^n$ in the form of type-1 fuzzy sets defined on the domain of discourse $[0,1]$, an associated type-1 OWA operator of dimension n is a mapping Φ,

$$\begin{array}{rcl} \Phi\colon F(X)\times\cdots\times F(X) & \longrightarrow & F(X) \\ (A_1,\cdots,A_n) & \mapsto & G \end{array}$$

that aggregates type-1 fuzzy sets $\{A_i\}_{i=1}^n$ in the following way,

$$\mu_G(y) = \sup_{\substack{\sum_{k=1}^{n} \bar{w}_i a_{\sigma(i)} = y \\ w_i \in U,\, a_i \in X}} \begin{pmatrix} \mu_{W_1}(w_1) * \cdots * \mu_{W_n}(w_n) \\ * \mu_{A_1}(a_1) * \cdots * \mu_{A_n}(a_n) \end{pmatrix} \tag{8}$$

where $*$ is a t-norm operator, $\bar{w}_i = \frac{w_i}{\sum_{i=1}^{n} w_i}$, and $\sigma : \{\ 1,\cdots,n\ \} \rightarrow \{\ 1,\cdots,n\ \}$ a permutation function such that $a_{\sigma(i)}$ is the i-th largest element in the set $\{a_1,\cdots,a_n\}$.

From the above definition, it can be seen that $\Phi(A_1,\cdots,A_n) = G \in F(X)$ is a type-1 fuzzy set defined on X, but Φ depends on the choice of the linguistic weights $\{W_i\}_{i=1}^{n}$. Hence Φ can be denoted as $\Phi_{W_1,\cdots,W_n}$. A Direct Approach indicated as follows can be used to perform type-1 OWA operation [8]:

Step 1:Initialisation.

- Given the set of linguistic weights $\{W_i\}_{i=1}^{n} \subseteq F(U)$ for aggregating the set of type-1 fuzzy sets $\{A_i\}_{i=1}^{n} \subseteq F(X)$;
- Given the discretised domains of the linguistic weights, $\hat{U}$, and that of the aggregated objects, $\hat{X}$;
- Let the initial aggregation result $\overline{G} = (\overline{X},\ \mu_{\overline{G}})$, where $\overline{X} = \{0\}$, and $\mu_{\overline{G}}(\bar{x}) = 0$.

Step 2:Obtain the initial result $\overline{G}$

1) Select $w_1,\cdots,w_n \in \hat{U}$, $a_1,\cdots,a_n \in \hat{X}$;
2) Normalise $(w_1,\cdots,w_n)$ as $\bar{w}_i = \frac{w_i}{\sum_{i=1}^{n} w_i}$;
3) Perform the traditional OWA operation: $\bar{y} = \phi_{(\bar{w}_1,\cdots,\bar{w}_n)}(a_1,\cdots,a_n)$;
4) Calculate $\mu_0 = \mu_{W_1}(w_1) * \cdots * \mu_{W_n}(w_n) * \mu_{A_1}(a_1) * \cdots * \mu_{A_n}(a_n)$;
5) If there exists $y_k \in \overline{X}$ such that $\bar{y} = y_k$, update the potential membership grade $\mu_{\overline{G}}(y_k)$:
$$\mu_{\overline{G}}(y_k) \leftarrow max\left(\mu_{\overline{G}}(y_k), \mu_0\right)$$
Otherwise, $\bar{y}$ is added to $\overline{X}$, and $\mu_{\overline{G}}(\bar{y}) \triangleq \mu_0$;
6) Go to ***Step 2***-(1), and continue until all weight vectors and aggregating points are selected.

Step 3:Derive the fuzzy set G on $\hat{X}$:

$$\mu_G(\hat{x}) = \sup_{\bar{x}_j \in \Theta_{\hat{x}}} \left(\mu_{\overline{G}}(\bar{x}_j)\right).$$

Figure 2 shows an example of aggregating four fuzzy sets by type-1 OWA operator using the linguistic weights in Figure 1.

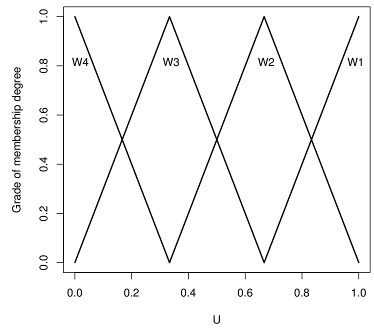

Fig. 1: Linguistic weights used in type-1 OWA (from right to left): W_1, W_2, W_3 and W_4

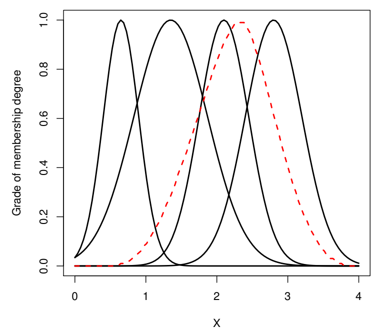

Fig. 2: Aggregation of type-1 OWA operator with linguistic weights given in Figure 1- solid lines: fuzzy sets to be aggregated; dashed line: aggregation result

IV. A Type-1 OWA Based Non-Stationary Fuzzy System

Different from the standard fuzzy systems, a non-stationary fuzzy system is a collection of n repeatedly running normal fuzzy inferences, in which the antecedent and consequent parts and each rule inference output are fuzzy sets. Each time the fuzzy inference process uses an instantiation of the non-stationary fuzzy set. Hence, n output sets are generated after running through the non-stationary reasoning. As a result, in a non-stationary fuzzy system some additional components become necessary besides the commonly used fuzzifier, rule base, rule engine, defuzzifier. Among them, an important additional component is to aggregate these n output sets into an overall one. In this paper, we propose to apply the type-1 OWA operator as uncertain operator to aggregate the output sets, which leads to a type-1 OWA based non-stationary fuzzy system (T1ONFS) as depicted in Figure 3.

Generally speaking, the T1ONFS works as follows. In each run, crisp input values first feed into the system through the fuzzifier by which the fuzzification of these inputs is carried out in a singleton or non-singleton way. The fuzzified non-stationary fuzzy sets then activate the inference engine and rule base to yield output set by performing the union and intersection operations of fuzzy set and compositions of relations. This process repeats n times. So n output sets are generated. Then a type-1 OWA operator is applied to these output sets to generate an overall set by combining these output sets. Finally, this overall fuzzy set is defuzzified to produce a crisp output.

Specifically speaking, consider a T1ONFS fuzzy system having m inputs $x_1 \in X_1,\cdots,x_m \in X_m$ and one output $y \in Y$, the rule base contains L non-stationary fuzzy rules expressed in the following form:

$$\dot{R}_t^l:\ if\ x_1\ is\ \dot{F}_{1,t}^l\ and\ \cdots and\ x_m\ is\ \dot{F}_{m,t}^l,\ then\ y\ is\ \dot{G}_t^l \qquad (9)$$

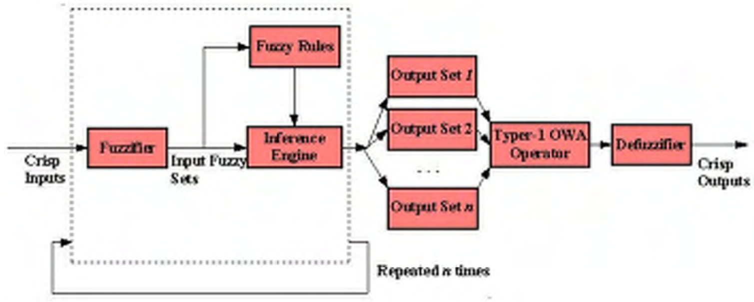

Fig. 3: Type-1 OWA based non-stationary fuzzy system

where $l = 1, \cdots, L$, $\dot{F}_{i,t}^{l}$ and $\dot{G}_{t}^{l}$ are non-stationary fuzzy sets, $t = 1, \cdots, n$ (n represents the number of repetitions). These rules represents non-stationary fuzzy relations between the multiple dimensional input space $X \triangleq X_1 \times \cdots \times X_n$ and the output space Y. In each run, given an input x, the singleton or non-singleton fuzzifier can be used to maps $x$into a fuzzy set $\dot{A}_x$. The inference engine combines the above rules and maps $\dot{A}_x$ into an consequent set $\dot{B}_t^l$ by using the extended sup-star composition principle,

$$\mu_{\dot{B}_t^l}(y) = \mu_{\dot{A}_x \circ \dot{R}_t^l}(y) = \cup_{x \in X}\left[\mu_{\dot{A}_x}(x) \cap \mu_{\dot{R}_t^l}(x,y)\right] \tag{10}$$

where $\cup$ and $\cap$ are union and intersection operators respectively, and

$$\mu_{\dot{R}_t^l}(x,y) = \left[\cap_{i=1}^{m} \mu_{\dot{F}_{i,t}^l}(x)\right] \cap \mu_{\dot{G}_t^l}(y) \tag{11}$$

Then the union of these fuzzy sets leads to an output set:

$$\mu_{B_t}(y) = \cup_{l=1}^{L} \mu_{\dot{B}_t^l}(y) \tag{12}$$

This process repeats until $t = n$. As a result, n output sets $B_t(t = 1, \cdots n)$ are produced. Then a type-1 OWA operator, $\Phi_{W_1,\cdots,W_n}$, is used to aggregate these $B_t(t = 1, \cdots n)$ into an overall output set B:

$$B = \Phi_{W_1,\cdots,W_n}(B_1, \cdots, B_n) \tag{13}$$

where $W_1, \cdots, W_n$ are the associated linguistic weights. Finally, a crisp output y_0 is obtained by defuzzfying the set B.

In our study of designing a non-stationary fuzzy expert system for breast cancer treatments, 12 initial fuzzy rules are acquired [11] according to the professional clinical guidelines provided by Nottingham University Hospitals NHS Trust Breast Directorate, i.e., the fuzzy rule base is obtained from human experts' knowledge, which is different from the scheme of inducing fuzzy rules from a dataset [9] [10]. These guidelines include various treatment decisions based on many patients' assessment results (corresponding to the attributes described in the Section II). The guidelines for chemotherapy treatment used in this paper is shown in Table I.

TABLE I: Clinical guideline for adjuvant system therapy

Conditions	***Recommendations***
NPI < 3.0	No Adjuvant Treatment
NPI 3.1-3.4	
ER + positive	Recommend Hormone therapy
ER - negative	Recommend Chemotherapy if VI
NPI 3.4-4.4	
ER + positive	Recommend Hormone therapy
ER - negative	Recommend Chemotherapy
NPI > 4.4	
ER + positive	Discuss Chemotherapy
	Recommending Chemotherapy:
	Age < 40
	VI
	HER-2 +positive
	Weak ER (<100/300)
	Recommending Against Chemotherapy:
	Age > 60
	Only 1 LN positive
	Special type cancer
ER - negative	Recommend Chemotherapy

In the Table I, seven attributes: *NPI*, *ER*, *Age*, '*HER-2*', *VI*, *LN* and '*Special type cancer*' information are considered by the multi-disciplinary team in deciding the treatment recommendations. However, both '*HER-2*' and '*Special type cancer*' content is neither available nor clearly described in the given dataset. For this reason, only the other five features are used as the inputs. Meanwhile, from the given adjuvant therapy guidance, it can be seen that most of decisions are related to chemotherapy, hence the fuzzy system output is set to be a particular 'chemotherapy recommendation decision'. So three decisions: C_1='Yes', C_2='No' and C_3 'Maybe' are considered to be outcomes, which correspond to the guideline decisions: '*recommend chemotherapy*', '*against chemotherapy*' and '*maybe recommend chemotherapy*' respectively. If the recommendation is '*no adjuvant treatment*' and '*recommend hormone therapy*', it will be considered as '*against chemotherapy*' in this study.

So given the state of a patient, the proposed T1ONFS is applied to the medical diagnosis for this patient, the corresponding crisp output y_0 is obtained. Then the treatment recommendation is made according to the principle of maximal membership:

$$i_0 = \arg \max_{1 \le i \le 3} (C_i(y_0)) \tag{14}$$

V. Experimental Results

In this section, we apply the proposed fuzzy expert system to the breast cancer clinical data collected from Nottingham

City Hospital, in which 1310 breast cancer cases are considered. Each cancer case is to be diagnosed by the non-stationary fuzzy system that runs 10 times, then the diagnosis result is to be compared with the doctor's recommendations. The system performance will be evaluated in terms of the rate of agreement with the doctor's judgments. Also, the proposed method will further compare with the fuzzy weighted averaging (FWA) operator [12], a widely used aggregation operator in combining different fuzzy sets [13], [14].

It can be seen from the Definition 2 that type-1 OWA operators depend on the choices of linguistic weights $\{W_i\}_{i=1}^{10}$ [8]. Different choices of linguistic weights may lead to different type-1 OWA operators. In this study, the meet-like type-1 OWA operator and join-like type-1 OWA operator will be chosen.

Meet and join are two operators working on fuzzy sets first proposed by Zadeh [15], named in [16], and widely investigated recently [17], [18]. Interestingly, a meet-like type-1 OWA operator can be obtained by selecting appropriate linguistic weights: the last linguistic weight approaching to the singleton fuzzy set $\tilde{1}$, and the rest of linguistic weights approaching to $\tilde{0}$ in turn. One example of a meet-like type-1 OWA operator operator is to choose the linguistic weights $\{W_i\}_{i=1}^{10}$ as depicted in Figure 4, in which $W_{10} = A_1$ and $W_i = A_0$, $i \neq 10$. We denote this meet-like type-1 OWA operator as MLT1OWA.

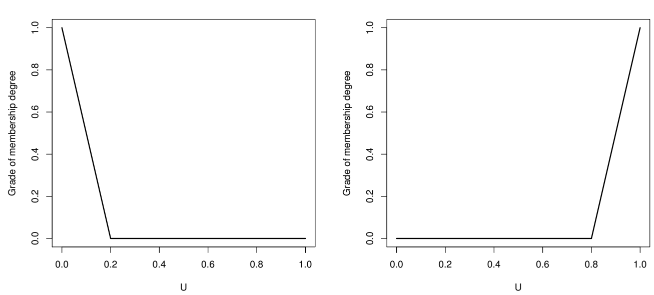

Fig. 4: Linguistic weights: (left) A_0; (right) A_1

Also, a join-like type-1 OWA operation can be obtained by selecting appropriate linguistic weights. Indeed, this is the case when the first linguistic weight is close to the singleton fuzzy set $\tilde{1}$, and the rest are close to the singleton fuzzy set $\tilde{0}$ in turn. For example, the associated linguistic weights $\{W_i\}_{i=1}^{10}$ chosen as $W_1 = A_1$ and $W_i = A_0$, $i \neq 1$ (depicted in Figure 4) leads to a join-like type-1 OWA operator, which is denoted as JLT1OWA.

It is known that the crisp mean operator has weights all equal to $1/n$. FWA operator with the weights approaching to singleton fuzzy sets $\widetilde{1/n}$ can be seen as the extended mean operation on fuzzy sets. In this study, a FWA with equal linguistic weights in the forms of triangular fuzzy numbers whose cores locate at 0.1 will be chosen.

Then we apply the non-stationary fuzzy systems with different aggregation operators as chosen above to the 1310 breast cancer cases, and the treatment decisions are made according to the principle of maximal memberships. Table II, Table III and Table IV are the confusion matrices of the agreements of the different aggregation operators based non-stationary fuzzy systems with doctor's judgments, in which the MLT1OWA, JLT1OWA and FWA based non-stationary fuzzy systems are used to provide soft decision supports for breast cancer treatments respectively. It can be seen that the non-stationary fuzzy system with a meet-like type-1 OWA operator can achieve better performance, but the choice of type-1 OWA operators for a non-stationary fuzzy system to achieve the best classification performance is an unresolved problem.

TABLE II: Confusion matrix obtained by MLT1OWA based fuzzy decision

Confusion Matrix		Clinician Decision		
		No	Maybe	Yes
Model Decision	No	79%	4.1%	14.6%
	Maybe	0.2%	0.0%	0.0%
	Yes	1.8%	0.0%	0.3%

TABLE III: Confusion matrix obtained by JLT1OWA based fuzzy decision

Confusion Matrix		Clinician Decision		
		No	Maybe	Yes
Model Decision	No	57%	3.1%	9.7%
	Maybe	1.4%	0.1%	0.2%
	Yes	22.6%	0.8%	5.0%

VI. Conclusions

In this paper, in order to model expert's variations in automated decision supports for breast cancer treatments, the type-1 OWA based non-stationary fuzzy system has been proposed, in which the multiple runs fuzzy sets are aggregated into an overall one by using the uncertain OWA operator. The proposed scheme provides a flexible way of choosing linguistic weights for non-stationary fuzzy system in combining expert's variations. We believe the proposed scheme would lead some new issues worthy to be investigated, for example, how to identify optimal linguistic weights in non-stationary fuzzy systems? We will pay attention to this topic in the future.

Acknowledgment

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TABLE IV: Confusion matrix obtained by FWA based fuzzy decision

Confusion Matrix		Clinician Decision		
		No	Maybe	Yes
Model Decision	No	75%	3.8%	13.9%
	Maybe	1.6%	0.0%	0.2%
	Yes	4.5%	0.3%	0.8%

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