

Preliminary Investigations into Modelling the Variation in Human Decision Making

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Abstract

This paper presents preliminary investigations into modelling the variation in human decision making. The relationship between the uncertainty introduced to the membership functions (mfs) of a Fuzzy Logic System (FLS) and the variation in decision making is explored using two separate methods. Initially uncertainty is introduced to a type-1 FLS by adding noise to its mfs and the effect on decision making is examined. Secondly an interval type-2 FLS is developed by representing the terms used in the FLS with interval type-2 fuzzy sets and the variation in decision making is studied using the FLS's interval outputs. The variations in ranking of umbilical acid-base assessments by six experts is compared to the simulation results from the developed FLSs. It is shown that there is a direct relationship between the variation in decision making and the uncertainty in the linguistic terms used, and the level of variation is proportional to the magnitude of uncertainty.

Keywords: nondeterministic fuzzy reasoning, type-2 fuzzy logic systems, umbilical acid-base assessment

1 Introduction

Uncertainty pervades all decision-making and appears in a number of different forms. The gen-

eral framework of fuzzy reasoning allows handling of much of this uncertainty. Fuzzy systems usually employ type-1 fuzzy sets, which represent uncertainty by numbers in the range $[0,1]$. These numbers stand for the degree of membership, certainty on whether the corresponding elements belong to the set or not. Type-2 fuzzy sets are an extension to this where an additional dimension corresponds to the uncertainty of the degree of membership. For example, consider the type-2 set of young people; whether age of 25 belongs to this set or not may have a membership of 0.8 with a certainty of 0.6 and a membership of 0.9 with a certainty of 0.5. Type-2 fuzzy sets are useful in circumstances where it is difficult to determine the exact mf for a fuzzy set. Type-1 mfs are totally precise, once they have been chosen all the uncertainty disappears. However, mfs of type-2 fuzzy sets are fuzzy themselves. The simplest kind of type-2 sets are interval type-2 sets whose elements' degree of membership are intervals with certainty of 1.0.

Fuzzy logic systems (FLSs) are used for representing and inferring with knowledge that is imprecise, uncertain, or unreliable. They consist of four main interconnected components: rules, fuzzifier, inference engine, and output processor. Once the rules have been established, an FLS can be viewed as a mapping from inputs to outputs. A typical rule is like:

IF arterial pH is *low* and venous pH is *low*
THEN acidemia is *severe*.

Fuzzy sets are associated with linguistic terms of the rules, shown in italics above, and with the inputs to and the output of FLS. Type-1 FLSs use type-1 fuzzy sets and an FLS which uses at

least one type-2 fuzzy set is called a type-2 FLS. Inferencing and output processing of a general type-2 FLS are prohibitive [1]. A simplification approach is to use interval type-2 fuzzy sets. There are fast algorithms to compute the output of an interval type-2 FLS (it2FLS) [1].

The concept of type-2 fuzzy sets was introduced by Zadeh [2]. Mizumoto and Tanaka studied the set theoretic operations of type-2 fuzzy sets and properties of membership degrees of such sets [3]; and examined type-2 fuzzy sets under the operations of algebraic product and algebraic sum [4]. Karnik and Mendel obtained algorithms for performing union, intersection, and complement for type-2 fuzzy sets, and developed the concept of the centroid of a type-2 fuzzy set [1]. Dubois and Prade gave a formula for the composition of type-2 relations as an extension of the type-1 sup-star composition for the minimum t-norm [5]. Karnik et al. presented a general formula for the extended sup-star composition of type-2 relations [6]. Hisdal studied rules and interval sets for higher-than-type-1 fuzzy logic [7]. Liang and Mendel developed the theory for different kinds of fuzzifiers for it2FLSs [8]. Mendel and John developed a simple method to derive union, intersection and complement of type-2 fuzzy sets without having to use Zadeh's extension principle [9].

Type-1 FLSs are deterministic in the sense that for the same inputs the outputs are always the same. However, human experts exhibit a nondeterministic behaviour in decision making. Variation may occur among the decisions of a panel of human experts as well as in the decisions of an individual expert for the same inputs. The terms that are used in an FLS have different meanings for different experts and experts may arrive to different conclusions in their inferencing depending on environmental conditions or over time. Understanding the dynamics of the variation in human decision making could allow the creation of 'truly intelligent' systems that cannot be differentiated from their human counterparts. Moreover, in application areas where having an expert constantly available is not possible, such systems can produce a span of decisions that may be arrived at by a panel of experts. The main aim of the work presented in this paper is to determine the parameters

that define the uncertainties resulting in variation in decision making.

Diagnostic medicine, where systematic handling of perceptual uncertainties is crucial to success, is an important application domain for this study. The relationship between the uncertain mfs used in the FLSs and the variation in decision making is explored using two different FLSs developed for umbilical acid-base (UAB) assessment. UAB assessment of an infant immediately after delivery is an objective measure of labour, and can be used to audit assessment of labour performance. The acidity (pH), partial pressure of oxygen (pO_2) and partial pressure of carbon dioxide (pCO_2) in blood samples taken from the venous and arterial vessels in the clamped umbilical cord can be measured by a blood gas analysis machine. A parameter termed base deficit of extracellular fluid (BDecf) can be derived from the pH and pCO_2 parameters [10]. An interpretation is made based on the pH and BDecf parameters from both arterial and venous blood. A type-1 FLS was previously developed for the UAB assessment, encapsulating the knowledge of leading obstetricians, neonatologists and physiologists gained over years of acid-base interpretation [11–14]. This FLS combines knowledge of the errors likely to occur in acid-base measurement, physiological knowledge of plausible results and statistical knowledge of a large database of results.

The FLSs developed to carry out the research presented in this paper are extensions of the original type-1 FLS. At first, a nondeterministic type-1 FLS (nd1FLS) was developed by adding noise to the type-1 mfs. Secondly, an it2FLS was developed by representing the terms used in the FLS by interval type-2 fuzzy sets. The details of these FLSs are explained further in the next section. The variation in decision making is examined in terms of the magnitude of the uncertainty introduced to the original type-1 mfs. The results of the study are presented in section 3. The paper concludes with discussions of the results and an outline of future work in section 4.

2 Methodology

2.1 The Original FLS

The original type-1 FLS is based on four main input variables; arterial pH (pH_A), arterial base deficit (BD_A), venous pH (pH_V), and venous base deficit (BD_V). Each of the four input parameters is assigned a linguistic variable and each is divided into three fuzzy terms, corresponding to meanings of *low*, *medium* and *high*. Three output fuzzy variables are used, severity of acidemia (*acidemia*), duration of acidemia (*duration*), and component of the acidemia (*component*). The acidemia variable has six terms in its term-set: *severe*, *significant*, *moderate*, *mild*, *normal* and *alkalotic*; the duration variable has three terms: *chronic*, *intermediate* and *acute* and the component variable has three terms in its term-set: *metabolic*, *mixed* and *respiratory*.

The rules for the FLS are obtained as a result of knowledge elicitation sessions with several leading clinicians skilled in umbilical cord blood acid-base analysis, and has been carefully refined to form a complete and consistent set of classifiers. A synopsis of the fuzzy rule set is shown in Table 1. The symbol ‘—’ indicates any value of the parameter in the context of input variables, and no consequence result in the context of output variables - i.e. the variable is not utilised in the rule. The *very* hedge was taken as the square operator.

The Mamdani model of inference is used. The probabilistic family of operators is chosen, in which conjunction is defined as $(a * b)$, disjunction as $(a + b - a * b)$ and negation as $1 - a$. The min operator is used for implication. Centre-of-gravity defuzzification is used to produce crisp values for each fuzzy output variable.

2.2 Variation in Experts’ Views

Six expert clinicians who took part in the development of the type-1 FLS were asked to rank 50 UAB assessments in terms of perceived likelihood of having suffered brain damage due to lack of oxygen. Fig. 1 shows the rankings of 50 UAB assessments by six experts against the type-1 FLS. A perfect agreement, which would be a straight line from (0,0) to (50,50), is the ideal desired result. However, as can be seen from

Table 1: a synopsis of the rule set

Rule	Input Variables				Output Variables		
	pH_A	BD_A	pH_V	BD_V	<i>acidemia</i>	<i>component</i>	<i>duration</i>
1	low	high	low	high	very severe	metabolic	very chronic
2	low	high	low	mid	severe	metabolic	chronic
3	low	high	low	low	severe	metabolic	intermediate
4	low	high	mid	—	severe	metabolic	intermediate
5	low	high	normal	high	severe	metabolic	intermediate
6	low	high	normal	not high	severe	metabolic	acute
7	low	mid	low	not low	significant	mixed	chronic
8	low	mid	low	low	significant	mixed	intermediate
9	low	mid	mid	—	significant	mixed	intermediate
10	low	mid	normal	high	significant	mixed	acute
11	low	mid	normal	not high	significant	mixed	acute
12	low	low	low	not high	moderate	respiratory	chronic
13	low	low	mid	not high	moderate	respiratory	intermediate
14	low	low	normal	not high	moderate	respiratory	acute
15	mid	high	mid	not low	moderate	metabolic	chronic
16	mid	high	mid	low	moderate	metabolic	intermediate
17	mid	high	normal	—	moderate	metabolic	acute
18	mid	mid	mid	high	mild	mixed	chronic
19	mid	mid	mid	not high	mild	mixed	intermediate
20	mid	mid	normal	high	mild	mixed	intermediate
21	mid	mid	normal	not high	mild	mixed	acute
22	mid	low	mid	not high	mild	respiratory	intermediate
23	mid	low	normal	not high	mild	respiratory	—
24	normal	—	normal	—	normal	—	—
25	high	—	high	—	alkalotic	—	—

Fig. 1, there is neither perfect agreement with the FLS nor among the experts. It can also be observed that at the extreme cases the experts tend to agree with each other and the FLS but in the cases that fall in the middle of the range, there is less agreement. The distribution presents the characteristic of an elliptic envelope around the diagonal line from (0,0) to (50,50).

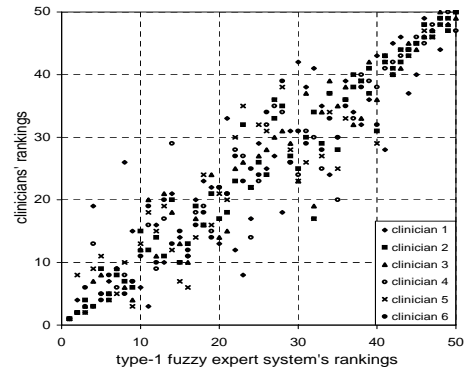


Figure 1: Ranking Variation of 50 UABs

The aim of this research is to explore the dynamics of the variation in human decision making. In this paper, the relation between the uncertainty in mfs used by the FLSs and the variation in decision making is studied. The rules of the type-1 FLS use fixed type-1 mfs to represent linguistic terms. However, experts have diverse opinions about meanings of linguistic terms and they often provide different consequents for the same antecedents of a rule. In this study the vagueness inherent in the linguistic labels was in-

roduced by two approaches which are explained in the following sections.

2.3 Nondeterministic Type-1 FLS (nd1FLS)

There are many ways in which vagueness might be introduced into the mfs of an FLS. An original approach proposed in this study is to blur the type-1 mfs by deviating the parameters defining the membership function in some way at each evaluation. The terms used in the rule base of the original type-1 FLS have mfs based on sigmoidal functions. In the new FLS the mfs have uncertain centres where the centre point is shifted from left to right by an amount randomly distributed over a specified fraction of the universe of discourse. So, an alteration in the centre of 1% of the universe of discourse is referred to as *noise level* of 1%. In the experiments carried out, the noise level was varied from 0.1% to 1%. All other components of the FLS remain the same (i.e. fuzzifier, output processor, etc) apart from inferencing which is modified slightly so that the mfs are updated each time they are referred to. Other variations of this structure could be built by updating the mfs either each time the FLS is started or whenever a rule is evaluated. The resulting FLS is nondeterministic in the sense that for the same input the output is different each time the FLS is run, so it is referred to as a nondeterministic FLS (nd1FLS) in this paper.

The inferencing module updates the mfs each time they are referred to, so instead of enumerating every possible membership functions only one of them is chosen each time. The uncertain nature of the mfs of nd1FLS reminds characteristics of type-2 mfs. In a general type-2 FLS, most operations are prohibitive because it is necessary to enumerate all the embedded mfs within the type-2 mf. In nd1FLSs, each time an mf is updated, the resulting mf is type-1 and remaining operations are only carried out for this single type-1 mf. The results of the experiments for analyzing the effect of uncertainty in mfs on variation in decision making is presented in the section 3.1. In these experiments the nd1FLSs were run 100 times to rank 50 UAB assessments.

2.4 Interval Type-2 FLS (it2FLS)

Using a type-2 FLS can effectively provide a natural mechanism to present the vagueness inherent in linguistic terms used in FLSs.

In the second stage of the work presented in this paper the type-1 FLS was extended by converting the rule set directly by using interval type-2 fuzzy sets. In the it2FLS, the uncertainty about the shape of the original type-1 sigmoidal mfs is introduced by assuming a 1-5% shift in their centre points which results in interval type-2 fuzzy sets with sigmoidal primary mfs. Fig. 2 shows three interval type-2 sigmoidal mfs, where c_i and c'_i ($i = 1, 2, 3$) are the centre point pairs for each type-2 mf.

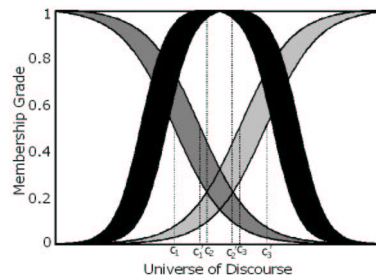


Figure 2: Three interval type-2 sigmoidal mfs

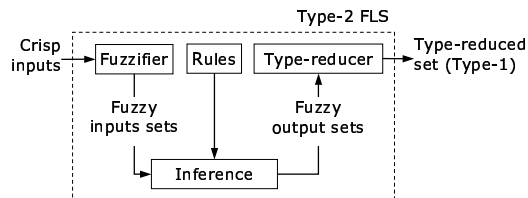


Figure 3: Interval type-2 FLS (it2FLS)

The components of the it2FLS developed for this study are presented in Fig. 3. The interference and defuzzification methods of the type-1 FLS were updated to work with the type-2 fuzzy terms. The fuzzifier of the type-1 FLS, which turns the crisp input values into type-1 fuzzy input sets in order to compensate for the errors in readings of the blood gas analysis machine, is not changed. By Mendel’s classification, the resulting FLS is a type-1 nonsingleton it2FLS because the inputs are type-1 fuzzy sets and all the antecedent and consequent sets of the rule base are interval type-2 fuzzy sets [1].

The input, antecedent, consequent and implication operations use minimum t-norm and maximum t-conorm. The result of antecedent opera-

tions is an interval which is fed into the consequent. Firing of rules result in type-2 fuzzy sets which are combined into a single type-2 fuzzy set by minimum t-norm. Mendel [1] has established theoretical results to effectively determine the lower and upper bounds of the centroid of a type-2 set and has provided algorithms [15] for carrying out the necessary calculations. That is, the centroid is an interval, the mean value of the upper and lower bounds of which can be taken as a single crisp centroid, if required.

A type-2 FLS must reduce to a type-1 FLS when the uncertainty about the shape of mfs is zero. This was verified by running the it2FLS with 0% deviation in the centre points of the mfs.

The type-1 FLS produces a health measure for every input case. These health measures are then used to rank the cases in terms of perceived likelihood of having suffered brain damage due to lack of oxygen. The health measure that is produced by the it2FLS is an interval. In order to demonstrate the effect of uncertainty in the linguistic terms of the rules, ranking is done for every combination of the cases using the upper and lower bounds of the health measure interval. This results in 2^{50} rankings for 50 UAB assessments. In the type-2 trials 20 UAB assessments are used because the storage requirements to keep the rankings for 2^{50} rankings is too high. The records of 2^{20} rankings requires 54 MB storage space so for 2^{50} rankings $\sim 2^{30} \times 54$ MB is necessary. Fig. 4 shows the variation in experts' rankings of the 20 UAB assessments used in these trials.

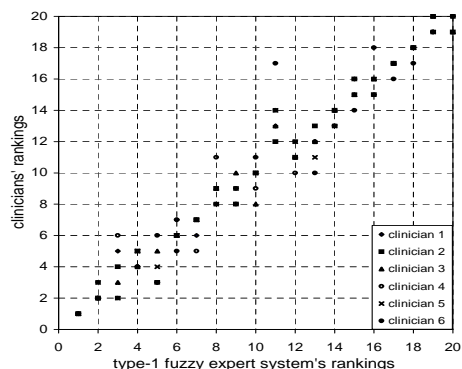


Figure 4: Ranking Variation of 20 UABs

Effects of the vagueness inherent in interval mfs on variation in decision making is presented in section 3.2.

2.5 Inferencing of the it2FLS

In the it2FLS the same set of operators are used as in the original FLS. The following theorem is used in implementing the inferencing of the it2FLS.

Theorem 2.1 [8] *In an interval type-1 nonsingleton type-2 FLS with meet under product or minimum t-norm: (a) the result of the input and antecedent operations, which are contained in a firing set is an interval type-1 set, i.e.*

$$F^l(x') = \left[\underline{f}^l(x'), \bar{f}^l(x') \right] \equiv \left[\underline{f}^l, \bar{f}^l \right]$$

where

$$\underline{f}^l(x') = \sup_x \int_{x_1 \in X_1} \dots \int_{x_p \in X_p} \left[\begin{array}{c} \left[\mu_{X_1}(x_1) * \underline{\mu}_{\bar{F}_1^l}(x_1) \right] * \\ \dots * \left[\mu_{X_p}(x_p) * \underline{\mu}_{\bar{F}_p^l}(x_p) \right] \end{array} \right] / x$$

and

$$\bar{f}^l(x') = \sup_x \int_{x_1 \in X_1} \dots \int_{x_p \in X_p} \left[\begin{array}{c} \left[\mu_{X_1}(x_1) * \bar{\mu}_{\bar{F}_1^l}(x_1) \right] * \\ \dots * \left[\mu_{X_p}(x_p) * \bar{\mu}_{\bar{F}_p^l}(x_p) \right] \end{array} \right] / x$$

and the supremum is attained when each term in brackets attains its supremum; (b) the rule R^l fired output consequent set, $\mu_{\bar{B}^l}(y)$, is a type-1 fuzzy set, where

$$\mu_{\bar{B}^l}(y) = \int_{b^l \in [\underline{f}^l(x') * \underline{\mu}_{\bar{G}^l}(y), \bar{f}^l(x') * \bar{\mu}_{\bar{G}^l}(y)]} 1/b^l, y \in Y$$

where $\underline{\mu}_{\bar{G}^l}(y)$ and $\bar{\mu}_{\bar{G}^l}(y)$ are the lower and upper membership grades of $\mu_{\bar{G}^l}(y)$; and, (c) suppose that N of the M rules in the FLS fire, where $N \leq M$, and the combined type-1 output fuzzy set is obtained by combining the fired output consequent sets; i.e., $\mu_{\bar{B}}(y) = \sqcup_{l=1}^N \mu_{\bar{B}^l}(y)$; then,

$$\mu_{\bar{B}}(y) = \int_{b^l \in \left[\begin{array}{c} [\underline{f}^1(x') * \underline{\mu}_{\bar{G}^1}(y)] \vee \dots \vee [\underline{f}^N(x') * \underline{\mu}_{\bar{G}^N}(y)] \\ [\bar{f}^1(x') * \bar{\mu}_{\bar{G}^1}(y)] \vee \dots \vee [\bar{f}^N(x') * \bar{\mu}_{\bar{G}^N}(y)] \end{array} \right]} 1/b^l$$

A simple illustration of how this theorem is used in inferencing of an interval nonsingleton type-2 FLS is shown in Fig. 11-2 on page 362 in [1].

3 Results

In this section, the effect of introducing uncertainty about the shape of the mfs of the linguistic terms used in the FLSs is presented and the effects of magnitude of the uncertainty on the variation is studied.

3.1 nd1FLSs

Fig. 5-9 show the three dimensional histograms of the simulation results of the nd1FLSs whose mfs are blurred by adding 0.1%-1% noise to the centre points of the sigmoidal functions. In the figures, the x-axis and y-axis correspond to the rankings of the deterministic and nondeterministic FLSs respectively and the z-axis shows the number of times the nd1FLSs produced the corresponding rank in the y-axis. In the experiments, nd1FLSs are used to rank 50 UAB assessments for 100 times. It can be observed that as the uncertainty about the linguistic terms used in the nd1FLSs is increased, the variation in decision making increases and the nd1FLSs often agree on ranks of the extreme cases but less frequently in the cases that fall in the middle of the range. Results present an elliptic envelop behaviour along the horizontal diagonal. As the amount of uncertainty is increased the envelop widens and number of agreements decreases.

3.2 it2FLS

Fig. 10-14 show the variation in rankings as the deviation in the centre point of the mfs used in the it2FLS is increased from 0.1% to 0.5%. It can be observed from these trials that there is a direct relationship between the uncertainty in the mfs used in the it2FLS and the variation in the rankings. As the uncertainty about the linguistic terms used in the it2FLS is increased, the variation in decision making is observed to increase. Another important observed feature is in the nature of the variation, the extreme UAB cases are ranked most of the time the same but in the cases that fall in the middle of the range, there is less agreement which results in a cloud of data bounded in an elliptic envelop along the diagonal. This is in parallel to the behaviour exhibited by the panel of experts presented in Fig. 1.

4 Discussions and Future Work

Most success of fuzzy logic is in fuzzy logic control, but this success has not yet been carried over to modelling human reasoning - Zadeh's Computing with words Paradigm [16]. In this paper, the work done on modelling the variation in human

expert opinion is presented. Specifically, the relation of the uncertainty in the mfs used in the FLSs and the variation in decision making is explored. It is shown that it is possible to capture the variation in human decision making using nd1FLS or it2FLSs. A nd1FLS is a type-1 FLS with noisy mfs which produces a different result for the same inputs each time it is run. In order to understand the general trends of the behaviour of the nd1FLSs, it is necessary to run them many times. In order to derive definite conclusions from this research, a it2FLSs was developed too. The simulation results confirm each other. It is observed that the level of variation is directly related to the amount of uncertainty about the linguistic terms used in decision making. The variation in decision making by an FLS can be controlled using the level of uncertainty in its mfs. This can be used in creating intelligent systems that can mimic their human counterparts better. An example of the major benefits that this may provide may be in application areas where having an expert constantly available is not possible. Such systems can produce a span of decisions that may be arrived at by a panel of experts.

The research on understanding and modelling the dynamics of variation in human decision making is ongoing. In the future work, the effect of different uncertainty sources on the variation in decision making will be explored. The immediate targets of this work is to overcome the hardware problems and increase the number of rankings that it2FLSs can handle to 2^{50} , and to examine the effects of introducing uncertainty to the mfs using different methods (e.g. deviating the width of the sigmoidal functions).

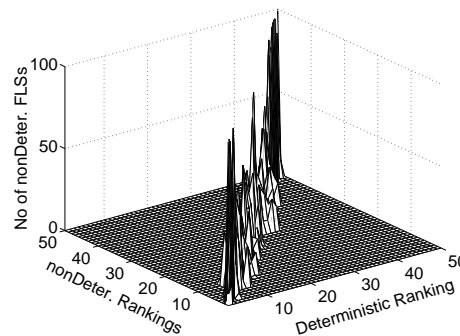


Figure 5: 0.1% deviation of the centre points

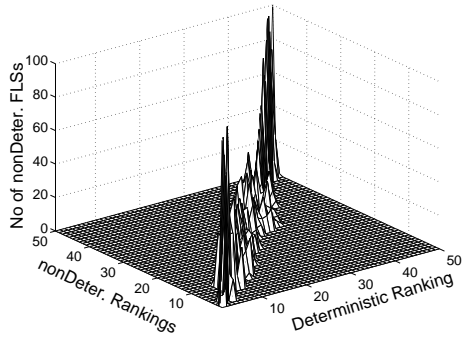


Figure 6: 0.13% deviation of the centre points

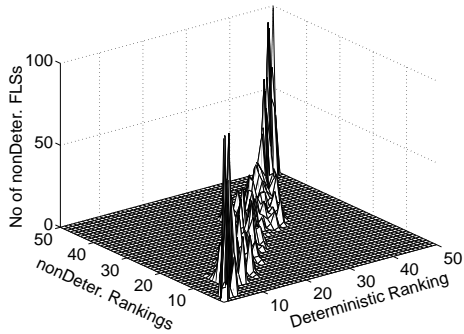


Figure 7: 0.2% deviation of the centre points

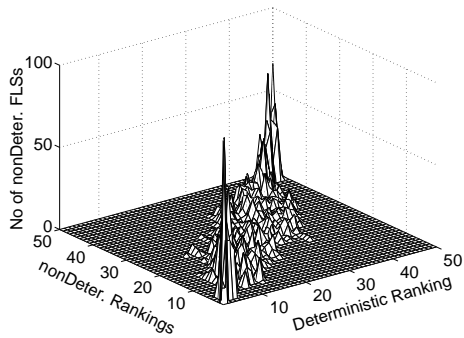


Figure 8: 0.4% deviation of the centre points

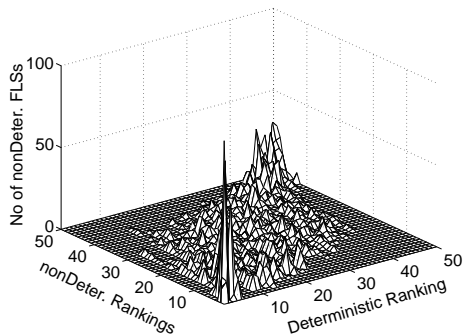


Figure 9: 1% deviation of the centre points

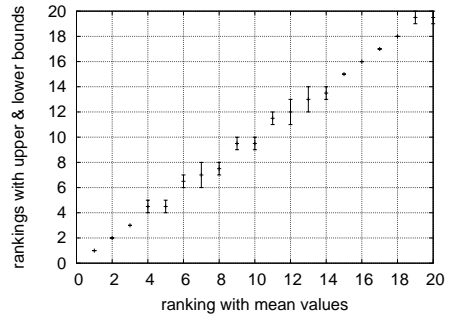


Figure 10: 0.1% deviation of the centre points

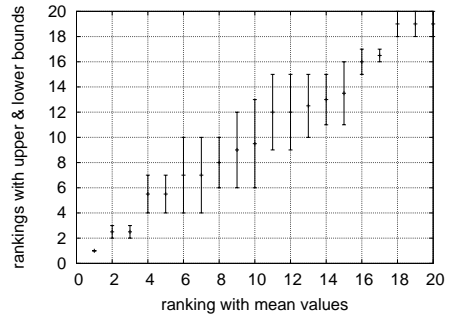


Figure 11: 0.2% deviation of the centre points

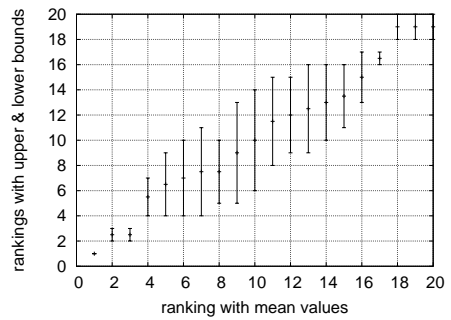


Figure 12: 0.3% deviation of the centre points

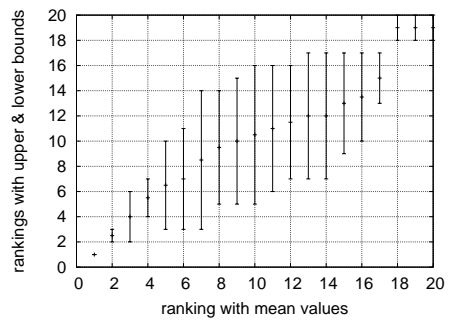


Figure 13: 0.4% deviation of the centre points

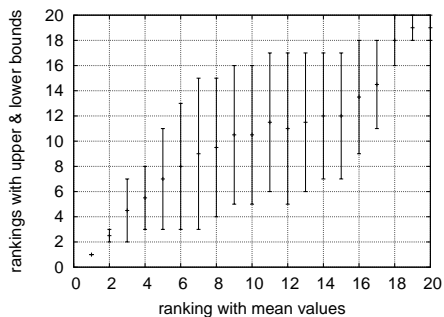


Figure 14: 0.5% deviation of the centre points

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