The Association between Non-Stationary and Interval Type-2 Fuzzy Sets: A Case Study

Jonathan M. Garibaldi, Salang Musikasuwan and Turhan Ozen
Automated Scheduling, Optimisation and Planning Group
University of Nottingham, Nottingham, NG8 1BB, UK
E-mail: {jmg, svm, txo}@cs.nott.ac.uk

Abstract— **In this paper a notion termed *non-stationary fuzzy sets* is introduced and the concept of random perturbations that can be used for generating these non-stationary fuzzy sets is also presented. A case study was carried out to investigate the relationship between the performance of non-stationary fuzzy logic systems and interval type-2 fuzzy logic systems. It can be observed that in case of centre variation, the lower-upper boundaries of outputs predicted by non-stationary systems are slightly narrower than those from the corresponding interval type-2 systems. On the other hand, in case of width variation, the lower-upper boundaries of outputs predicted by non-stationary systems are slightly wider than those from the type-2 systems. Moreover, an interesting observation is that the secondary membership function of the type-2 sets corresponding to non-stationary fuzzy sets generated using Normally distributed perturbations are non-uniform. In contrast to non-interval type-2 sets, this does not affect the inference process of the non-stationary sets. In this sense, the use of non-stationary fuzzy sets may enable approximations to be made of general type-2 fuzzy inferencing.**

I. Introduction

In 1965, Zadeh introduced the concept of fuzzy sets [1] in order to resemble human reasoning in its use of approximate information and uncertainty to generate decisions. They were specifically designed to represent uncertainty and vagueness and provided formalised tools for dealing with the imprecision in many real-world problems. Although knowledge can be expressed more naturally by using (type-1) fuzzy sets and many complex decision problems can be significantly simplified, type-1 fuzzy sets still have limitations, in that they are unable to model and minimize the effect of *all* uncertainties. Later, in 1975, Zadeh proposed 'fuzzy sets with fuzzy membership functions' as an extension of the concept of an ordinary, i.e. type-1, fuzzy set and went on to define fuzzy sets of type n, n = 2, 3, ..., for which the membership function ranges over fuzzy sets of type n - 1 [2]. Type-2 fuzzy sets can model uncertainties better and minimize their effects. The use of type-2 sets was advocated and extended by people: Dubois and Prade gave a formula for the composition of type-2 relations as an extension of the type-1 sup-star composition for the minimum t-norm [3], Mizumoto and Tanaka studied the set theoretic operations of type-2 sets and properties of membership degrees of such sets [4] and examined type-2 sets under the operations of algebraic product and algebraic sum [5], etc. However, their use in practice has been limited due to the significant increase in computational complexity involved in their implementation.

Recently, Mendel has established a set of terms to be used when working with type-2 fuzzy sets and, in particular, introduced a concept known as the *footprint of uncertainty* which provides a useful verbal and graphical description of the uncertainty captured by any given type-2 set. Mendel has particularly concentrated on a restricted class of general type-2 fuzzy sets known as *interval type-2 fuzzy sets* [6]. Interval type-2 sets are characterised by having secondary membership functions which only take the values 0 or 1. This restriction greatly simplifies the computational requirements involved in performing inference with type-2 sets. Mendel and John developed a simple method to derive union, intersection, and complement, and computational algorithms for type reduction (necessary for type-2 defuzzification) [7].

All humans, including *experts*, exhibit variation in their decision making. Variation may occur among the decisions of a panel of human experts (inter-expert variability), as well as in the decisions of an individual expert over time (intra-expert variability). Up to now it has been an implicit assumption that expert systems, including fuzzy expert systems (FESs), should not exhibit such variation. While type-2 sets capture the concept of introducing uncertainty into membership functions by introducing a range of membership values associated with each value of the base variable, they do not capture the notion of variability — a type-2 fuzzy inference system (FIS) will always produce the same output(s) (albeit a type-2 set with an implicit representation of uncertainty) for given the same input(s). Garibaldi et al [8]-[12] have been investigating the incorporation of variability into decision making in the context of FESs in medical domain. In this work, Garibaldi proposed the notion of 'non-deterministic fuzzy reasoning' in which variability is introduced into the membership functions of a fuzzy system through the use of random alterations to the parameters of the generating function(s). In this paper, this notion is extended and formalised through the introduction of a notion that we will term a 'non-stationary fuzzy set'. Finally, the relationship between non-stationarity and interval type-2 sets is explored through the case study in Section IV.

The rest of the paper is organised as follows. Section II gives a brief concept of type-2 fuzzy logic systems (FLS). Section III introduces the concept of non-stationary FLSs. The case study of using interval type-2 and non-stationary FLSs is presented in Section IV and the results in Section V. Finally, Section VI gives a discussion of the issues raised.

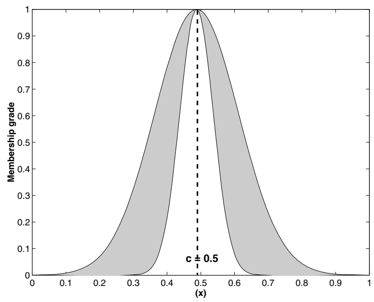

Fig. 1. Pictorial representation of a gaussian type-2 fuzzy set (width variation)

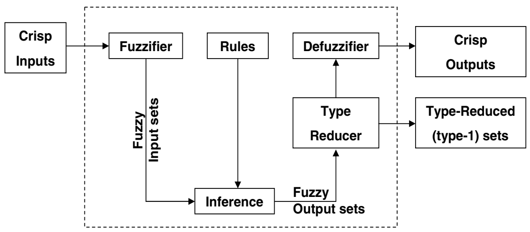

Fig. 2. Mechanisms of a type-2 FLS

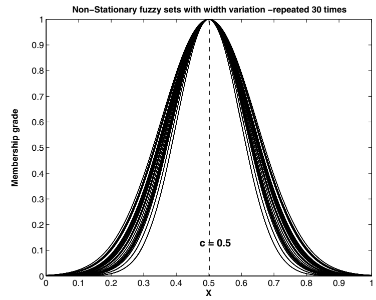

Fig. 3. Non-stationary fuzzy set (width variation) for which the set has been generated 30 times

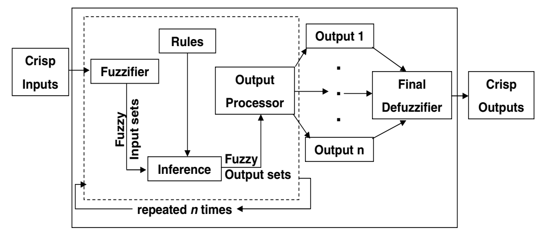

Fig. 4. Proposed mechanisms of a non-stationary FLS

II. Type-2 Fuzzy Logic Systems

All fuzzy sets are characterized by membership functions (MFs). Type-1 fuzzy sets are characterized by two-dimensional MFs in which each element of the type-1 fuzzy set has membership grade that is a crisp number in [0,1]. Type-2 fuzzy sets are characterized by fuzzy MFs that are three-dimensional. The membership grade for each element of a type-2 fuzzy set is a fuzzy set in [0,1]. The additional third dimension provides additional degrees of freedom to capture more information about the represented term. Type-2 fuzzy sets are useful in circumstances where it is difficult to determine the exact membership function for a fuzzy set, which is useful for incorporating uncertainties.

FLSs which are used for representing and inferring with knowledge that is imprecise, uncertain, or unreliable consist of four main interconnected components: *rules, fuzzifier, inference engine, and output processor*. Once the rules have been established, a FLS can be viewed as a mapping from inputs to outputs. Type-1 FLSs use only type-1 fuzzy sets and a FLS which uses at least one type-2 fuzzy set is called a type-2 FLS. The general type-2 fuzzy sets originally introduced by Zadeh [2] are too complicated, inferencing and output processing are prohibitive. A simplification approach is to use interval type-2 fuzzy sets in which there are fast algorithms to compute the output of an interval type-2 FLS [6]. Uncertainty in the primary memberships of a type-2 set consists of a bounded region that called the *footprint of uncertainty* (FOU), e.g. pictorial in Fig. 1. Each of the secondary membership functions of interval type-2 fuzzy sets has only one secondary grade that equals 1. Fig. 2 shows the mechanisms of a type-2 FLS. The interested reader is particularly referred to [6] for a summary tutorial and for more details.

III. Non-Stationary Fuzzy Logic Systems

As mentioned in Section I, Garibaldi proposed the notion of 'non-deterministic fuzzy reasoning' in which variability is introduced into the membership functions of a fuzzy system through the use of random alterations to the parameters of the generating function(s). In this section, this notion is extended and formalised through the introduction of a notion that we will term a *non-stationary fuzzy set*. The relationship between non-stationary fuzzy sets and type-2 fuzzy sets will be explored in Section IV (Case Study). Fig. 3 shows pictorial representation of non-stationary fuzzy set which has variation in width (standard deviation) and generated by repeating with random function for 30 times, where both initial centre and width (standard deviation) for fuzzy set equal to 0.5. Fig. 4 shows the mechanisms of how inferencing might be carried out using such non-stationary FLS's.

A. Non-Stationary Fuzzy Sets

Definition 1: A non-stationary fuzzy set, denoted $\dot{A}$, is characterised by a membership function, $\mu_{\dot{A}}(x,t)$, where $x \in X$ and $\mu_{\dot{A}}(x,t) \in [0,1]$ and t is a free variable, *time* — the time at which the fuzzy set is instantiated, i.e. as in Equation 1,

$$\dot{A} \;=\; \int_{x\in X} \mu_{\dot{A}}(x,t)\;\; x\;,\; \mu_{\dot{A}} \in [0,1] \qquad (1)$$

Any membership function may be used. In practice, of course only a few alternative membership functions are found in plain fuzzy sets, namely: piecewise linear including left-slope, triangular, right-slope, left-shoulder, trapeziodal, and right-shoulder; gaussian; and sigmoidal. The three main alternative kinds of non-stationary are:

- Variation in location
- Variation in slope
- Noise variation

There is no reason why non-stationary fuzzy sets should be Gaussian. For example, in Equation 2, let the universe of discourse, X, be the interval [1,2], with x interpreted as *height* (in metres). A non-stationary fuzzy set of X labeled *medium* (height) incorporating variability in the membership function might be represented by a Gaussian membership function where the width (standard deviation), σ, is a function of time:

$$medium(t) \quad = \quad \int_{1}^{2} e^{-\frac{(x-1.6)^2}{\sigma(t)^2}} \,/\, x \qquad (2)$$

At any given moment of time, i.e. in any specific instantiation, a non-stationary fuzzy set will instantiate a standard type-1 fuzzy set. Gaussian membership functions (MFs) have been used through out the rest of this study. In the context of Gaussian MFs, there are three main alternative kinds of non-stationarity:

- Variation in centre (Equation 3)
- Variation in width (Equation 4)
- Noise variation (Equation 5)

$$\mu(x,t) \quad = \quad e^{-\frac{(x-c(t))^2}{\sigma^2}} \qquad (3)$$

$$\mu(x,t) \quad = \quad e^{-\frac{(x-c)^2}{\sigma(t)^2}} \qquad (4)$$

$$\mu(x,t) \quad = \quad e^{-\frac{(x-c)^2}{\sigma^2}} \pm \varepsilon(t) \qquad (5)$$

B. Perturbation Functions

In Equation 3 - 5 above, $c(t)$, $\sigma(t)$, and $\varepsilon(t)$ can all be generated by using the following:

$$c(t) \quad = \quad c + kf(t) \qquad (6)$$

$$\sigma(t) \quad = \quad \sigma + kf(t) \qquad (7)$$

$$\varepsilon(t) \quad = \quad kf(t) \qquad (8)$$

where c and σ are the centre and width of the initially type-1 fuzzy set, respectively, k is a constant value, and $f(t)$ is the what we will term the *perturbation function*. By perturbation function, we mean a function (of time) that will generate small changes in the base membership function. In theory, this could be a true random function — i.e. the membership function parameter could be a true random variable: hence the terminology of *non-stationary* fuzzy sets.

In general, it would be appear that any function of time might be used as the perturbation function, where the only restriction is that membership function remains in bounds. Given that any measurement of time is arbitrary and relative, the actual set of functions that might be useful in practice is more restrictive. Any units might be used for time, t, but the most natural would be to express time in seconds (s), in the absence of any good reason not to. Again, given that any physical notion of time is relative, any arbitrary point in time might be chosen as zero. A few possibilities for perturbation functions in practice are:

- sine / cosine based, e.g.:

$$f(t) = sin(\omega t) \qquad (9)$$

- pseudo-random, e.g.:

$$s(t+1) = (25,214,903,917 s(t) + 11) \; mod \; 2^{48}$$
$$f(t) = \frac{s(t+1) - 2^{47}}{2^{47}} \qquad (10)$$

- differential time-series, e.g.:

$$f(x,t) \rightarrow \frac{dx(t)}{dt} = \frac{0\;2x(t-\tau)}{1+x^{10}(t-\tau)} - 0\;1x(t) \qquad (11)$$

IV. Case Study

In order to illustrate the use of non-stationary fuzzy sets and investigate the relationship between the performance of non-stationary FLSs and interval type-2 FLSs, this section focuses on constructing fuzzy systems to solve the XOR problem. Table I shows the classical XOR operation, where X and Y are input variables and Z is the output variable for those inputs.

In this study, FLSs were constructed to predict the output of truth value where both input variables can take any value in the range of [0,1]. The sets of input values are shown in Table II. All FLSs consist of two input variables which are *Input1* and *Input2*, one output variable which is *Output*, and four rules. Each variable consist of 2 Gaussian MFs which are *Low* and *High*. The following 4 rules are used for all FLSs. These rules are constructed based on the standard XOR problem.

1. IF *Input1* is *Low* AND *Input2* is *Low*
   THEN *Output* is *Low*
2. IF *Input1* is *Low* AND *Input2* is *High*
   THEN *Output* is *High*
3. IF *Input1* is *High* AND *Input2* is *Low*
   THEN *Output* is *High*
4. IF *Input1* is *High* AND *Input1* is *High*
   THEN *Output* is *Low*

TABLE I
XOR Truth Table

	X	Y	Z
Case 1	0	0	0
Case 2	0	1	1
Case 3	1	0	1
Case 4	1	1	0

TABLE II
INPUT VALUES FOR FUZZY SYSTEMS

Case	Input1	Input2	Output
Case 1	0.25	0.25	?
Case 2	0.25	0.75	?
Case 3	0.75	0.25	?
Case 4	0.75	0.75	?

A. *The Non-Stationary FLS*

There are three kinds of function of time that were used in this study, as follows:

- Sine function (Equation 9, where $\omega = 127$)
- Uniformly distributed pseudo-random function (Equation 10)
- Normally distributed random function (Matlab *randn* function)

In this paper, only the first two kinds of non-stationarity were investigated, i.e. *centre variation* and *width variation*. The first two functions above return numbers in the range $[-1. 1]$, while the third (the Matlab *randn* function) returns real numbers sampled from a Normal distribution with mean zero and standard deviation one. The non-stationary fuzzy sets were then generated by replacing *c(t)* or σ(t) in Equation 3 and 4 with $c(t) = c + 0\ 05 f(t)$ (Equation 6, where $k = 0\ 05$) and $\sigma(t) = \sigma + 0\ 05 f(t)$ (Equation 7, where $k = 0\ 05$), respectively. This process was repeated by the number of times given below.

Four different non-stationary FLSs for each of those three techniques (in total 12 non-stationary FLSs) have been designed with two inputs (antecedents), one output (consequent), two Gaussian MFs for each antecedents and consequent, and four rules. All terms (two inputs and one output) have two Gaussian membership functions, corresponding to meanings of *Low* and *High*. *Low* membership functions all have centre 0.1, *High* membership functions all have centre 0.9. Finally, the initial widths for all MFs for all terms are 0.5. The four different non-stationary FLSs are described by the number of repetitions used to construct the non-stationary MFs as follows:

- repeated 30 times
- repeated 100 times
- repeated 1,000 times
- repeated 10,000 times

B. *The Type-2 FLS*

Two interval type-2 FLSs have also been designed with 2 inputs (antecedents), 1 output (consequent), 2 Gaussian MFs for each antecedent and consequent, and four rules. The membership functions all have the same centre and width parameters as described above.

In the type-2 FLS, the footprint of uncertainty of the type-2 MFs are created by deviating the parameters of the original type-1 MFs by a percentage of the universe of discourse of the variables that they are associated to. Two different methods are used to create these type-2 MFs: by varying the centre point, and varying the width around the original type-1 MF. In the case of varying the centre, the centre of lower and upper bounds MFs were defined by shifting the initial centre point both left and right by a Δ amount, respectively, Where $\Delta (= 5\%)$ is a percentage of the universe of discourse of the variable that the MF belongs to, as following:

- Centre of lower MF = $c - \Delta$
- Centre of upper MF = $c + \Delta$

Similarly, in the case of varying the width, the width of lower and upper bounds MFs were defined by shifting the initial width both left and right by a Δ amount, respectively, as following:

- Width of lower MF = $\sigma - \Delta$
- Width of upper MF = $\sigma + \Delta$

TABLE III
DESCRIPTION OF ALL FLSs USED IN THIS PAPER

FLS	Description
it2FLS	Interval Type-2 FLS
nsFLS30N	30-repeats of non-stationary FLS with Normally distributed random numbers
nsFLS100N	100-repeats with Normally distributed random numbers
nsFLS1000N	1,000-repeats with Normally distributed random numbers
nsFLS10000N	10,000-repeats with Normally distributed random numbers
nsFLS30S	30-repeats of sine function with $t = 1 \ldots 30$
nsFLS100S	100-repeats of sine function
nsFLS1000S	1,000-repeats of sine function
nsFLS10000S	10,000-repeats of sine function
nsFLS30U	30-repeats with uniformly distributed random numbers
nsFLS100U	100-repeats with uniformly distributed random numbers
nsFLS1000U	1,000-repeats with uniformly distributed random numbers
nsFLS10000U	10,000-repeats with uniformly distributed random numbers

V. RESULTS

After all FLSs (13 each of both centre and width variations) had been constructed, the performance of all the systems was compared and the relationship between them analysed. Table III shows the description of all fuzzy models used in this paper. In case of centre variation, the lower and upper bounds predicted values and the final centroid output values for all 13 FLSs are shown in Table IV. For the non-stationary systems generated by Normally distributed random numbers (only), the lower and upper bounds are derived from $m \pm s$, where m is the mean of the outputs over time and s is the standard deviation. Similarly, the same information is presented for width variation in Table V.

TABLE IV
LOWER, MEAN AND UPPER BOUNDS FOR CENTRE VARIATION

	Input 1 (0.25,0.25)			*Input 2 (0.25,0.75)*			*Input 3 (0.75,0.25)*			*Input 4 (0.75,0.75)*		
FLS	*Lower*	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*
it2FLS	0.3687	0.3956	0.4224	0.5780	0.6050	0.6320	0.5780	0.6050	0.6320	0.3687	0.3956	0.4224
nsFLS30N	0.3829	0.3904	0.3979	0.5940	0.6023	0.6106	0.5960	0.6054	0.6148	0.3849	0.3937	0.4025
nsFLS100N	0.3842	0.3928	0.4014	0.5962	0.6046	0.6130	0.5953	0.6054	0.6155	0.3840	0.3943	0.4046
nsFLS1000N	0.3849	0.3936	0.4023	0.5965	0.6053	0.6141	0.5969	0.6060	0.6151	0.3851	0.3945	0.4039
nsFLS10000N	0.3850	0.3941	0.4032	0.5968	0.6057	0.6146	0.5968	0.6057	0.6146	0.3849	0.3941	0.4033
nsFLS30S	0.3853	0.3937	0.4024	0.5972	0.6067	0.6142	0.5969	0.6068	0.6139	0.3851	0.3938	0.4019
nsFLS100S	0.3850	0.3940	0.4028	0.5968	0.6069	0.6144	0.5969	0.6073	0.6144	0.3851	0.3944	0.4027
nsFLS1000S	0.3850	0.3937	0.4031	0.5967	0.6066	0.6146	0.5967	0.6063	0.6146	0.3850	0.3934	0.4031
nsFLS10000S	0.3850	0.3935	0.4031	0.5967	0.6064	0.6146	0.5967	0.6064	0.6146	0.3850	0.3935	0.4031
nsFLS30U	0.3853	0.3937	0.4024	0.5972	0.6067	0.6142	0.5969	0.6068	0.6139	0.3851	0.3938	0.4019
nsFLS100U	0.3856	0.3931	0.4032	0.5972	0.6060	0.6150	0.5973	0.6056	0.6146	0.3856	0.3927	0.4031
nsFLS1000U	0.3854	0.3934	0.4033	0.5969	0.6063	0.6150	0.5969	0.6066	0.6150	0.3854	0.3937	0.4033
nsFLS10000U	0.3850	0.3935	0.4031	0.5967	0.6064	0.6146	0.5967	0.6064	0.6146	0.3850	0.3935	0.4031

TABLE V
LOWER, MEAN AND UPPER BOUNDS FOR WIDTH VARIATION

	Input 1 (0.25,0.25)			*Input 2 (0.25,0.75)*			*Input 3 (0.75,0.25)*			*Input 4 (0.75,0.75)*		
FLS	*Lower*	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*	*Lower*	*Mean*	*Upper*
it2FLS	0.3836	0.3921	0.4007	0.5993	0.6079	0.6164	0.5993	0.6079	0.6164	0.3836	0.3921	0.4007
nsFLS30N	0.3716	0.3909	0.4102	0.5898	0.6091	0.6284	0.5986	0.6155	0.6324	0.3676	0.3845	0.4014
nsFLS100N	0.3697	0.3911	0.4125	0.5875	0.6089	0.6303	0.5932	0.6106	0.6280	0.3720	0.3894	0.4068
nsFLS1000N	0.3731	0.3922	0.4113	0.5887	0.6078	0.6303	0.5911	0.6098	0.6275	0.3725	0.3907	0.4089
nsFLS10000N	0.3730	0.3917	0.4104	0.5896	0.6083	0.6270	0.5898	0.6085	0.6272	0.3728	0.3915	0.4102
nsFLS30S	0.3742	0.3932	0.4088	0.5912	0.6068	0.6258	0.5920	0.6066	0.6264	0.3736	0.3934	0.4080
nsFLS100S	0.3734	0.3937	0.4093	0.5907	0.6063	0.6266	0.5908	0.6056	0.6264	0.3736	0.3944	0.4092
nsFLS1000S	0.3733	0.3931	0.4097	0.5903	0.6069	0.6267	0.5903	0.6075	0.6267	0.3733	0.3924	0.4097
nsFLS10000S	0.3732	0.3926	0.4098	0.5902	0.6074	0.6268	0.5902	0.6075	0.6268	0.3732	0.3925	0.4098
nsFLS30U	0.3736	0.3934	0.4080	0.5920	0.6066	0.6264	0.5912	0.6068	0.6258	0.3742	0.3932	0.4088
nsFLS100U	0.3736	0.3944	0.4092	0.5908	0.6056	0.6264	0.5907	0.6063	0.6266	0.3734	0.3937	0.4093
nsFLS1000U	0.3733	0.3925	0.4097	0.5903	0.6075	0.6267	0.5903	0.6069	0.6267	0.3733	0.3931	0.4097
nsFLS10000U	0.3732	0.3925	0.4098	0.5902	0.6075	0.6268	0.5902	0.6074	0.6268	0.3732	0.3926	0.4098

The non-stationary FLS with variation in centre was highlighted for further investigation. Given that the membership function is being varied non-uniformly, it is to be expected that the secondary membership functions of equivalent type-2 fuzzy sets are non intervals. In order to examine this, the membership functions were generated 10,000 times and the distribution of membership values in the secondary membership function was recorded. That is to say, the membership function is generated by horizontal Normal deviations (Normal deviations in the x-axis): the corresponding secondary function is the distribution in the y-axis. The distributions obtained in the y-axis at $x = 0\ 15$ and $x = 0\ 85$ for non-stationary membership functions with centre $c = 0\ 1$ and $c = 0\ 9$ respectively are shown in Fig. 5. The distribution of the final centroid output of the non-stationary FLS was also examined. The distributions of centroids obtained, again for 10,000 repeats, for input values of $(0\ 25, 0\ 25)$ and $(0\ 25, 0\ 75)$ are shown in Figs. 6 and 7.

VI. DISCUSSION

Examination of the results in Tables IV and V highlights a number of interesting observations. Firstly, the results for Inputs 1 and 2 are very similar to the results for Inputs 4 and 3, respectively. This applies to both centre and width variation and is entirely as expected from the symmetrical nature of the XOR problem.

For centre variation, the output interval of the type-2 FLS for Input 1 $(0\ 25, 0\ 25)$ is $[0\ 3687, 0\ 4224]$ with mean 0 3956. For the Normally distributed non-stationary FLS repeated 10,000 times, the corresponding interval is $[0\ 3850, 0\ 4032]$, with mean 0 3941 — i.e. the mean is similar, but the interval

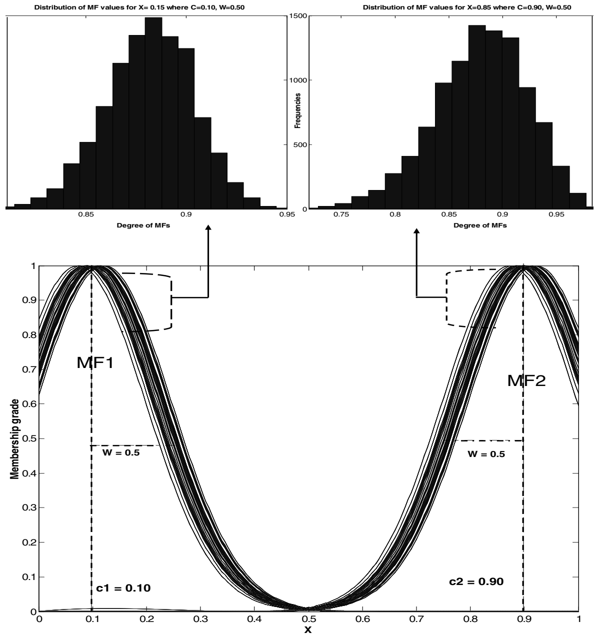

Fig. 5. Membership grades (nsFLS) for X1=0.15, X2=0.85, where m=0.10 and 0.90 and σ=0.50

is *narrower*. This finding is, in fact, repeated across all the results for centre variation. Similar results are observed for width variation except that in this case the intervals obtained are slightly *wider* than the corresponding type-2 intervals (the lower bounds are lower and the upper bounds are higher).

For non-stationary FLSs (i.e. those built on non-stationary fuzzy sets) utilising Normally distributed variations, the secondary membership functions of the corresponding type-2 FLSs are not uniform. In essence, simply by switching the underlying perturbation function from one utilising uniformly distributed values to one utilising Normally distributed numbers, the non-stationary FLS is in some sense simulating inferencing of a general type-2 FLS.

The simplest form of non-stationary fuzzy sets may be viewed as a generalisation of interval type-2 fuzzy sets. Clearly, more work needs to be done before any definitive claims can be made but, at this stage, it may be tentatively suggested that the use of non-stationary fuzzy sets may enable approximations to be made of general type-2 fuzzy inferencing. Research on understanding and modelling the dynamics of variation in human decision making is ongoing and the concepts of *non-stationary* fuzzy sets will be further explored.

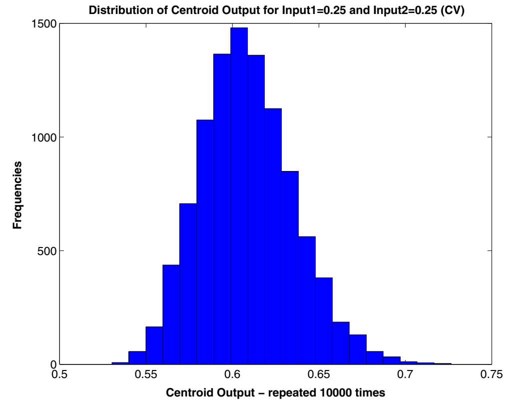

Fig. 6. Distribution of centroid output (nsFLS-centre variation) for Input1= Input2 = 0.25

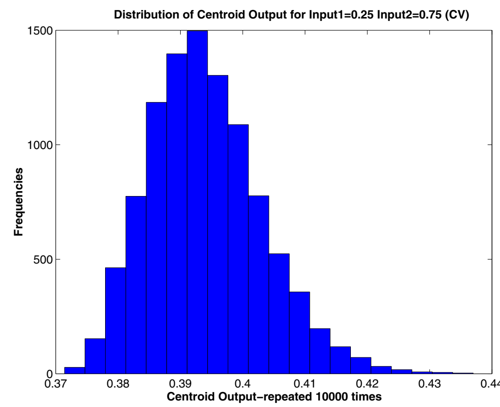

Fig. 7. Distribution of centroid output (nsFLS-centre variation) for Input1= 0.25 Input2 = 0.75

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