

# A Comparison of Type-1 and Type-2 Fuzzy Controllers in a Micro-Robot Context

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**Abstract**—In this paper we compare the differences between type-1 and interval type-2 fuzzy logic controllers, with seven, five and two three term membership functions. The controllers were used to control a DC motor model in a closed loop simulation. The performance of each controller to a step change and a change in motor inertia with and without added noise was recorded. The results showed that there was no statistical difference between the type-1 and type-2 controllers. It was also found that a type-1 three term controller was as good as a type-1 or type-2 seven term controller, in controlling the micro robot DC motor model.

## I. INTRODUCTION

This paper investigates type-1 and type-2 fuzzy logic controllers, as the basis for the control systems of soccer-playing micro robots. In 1975 Zadeh proposed ‘fuzzy sets with fuzzy membership functions’ as an extension of the fuzzy set. This produced fuzzy sets of *type-n*,  $n = 2, 3, \dots$ ; the membership function ranging over fuzzy sets of type  $n-1$  [1]. Mendel states that ‘Type-1 Fuzzy Logic Systems (FLSs) are unable to directly handle rule uncertainties, because they use type-1 fuzzy sets that are certain’ [2]. Type-2 FLSs use type-2 fuzzy sets which are described by membership functions which themselves are fuzzy. This allows Type-2 FLSs to model and handle the uncertainty of measurement and any rule uncertainty. Examples are the variability of expert opinion on a fuzzy set, and their self-referencing variability over time; opinions do change. Noise of the system and errors of measurement also have an effect [2].

However, some of the major problems with type-2 fuzzy sets are difficulty of understanding, envisaging how they look due to their 3-D nature, and the computational complexity needed to generate solutions. The derivations of union, intersection and complement all rely on the use of Zadeh’s Extension Principle [1]. Mendel and John [3] addressed these issues by ‘presenting a new representation for type-2 fuzzy sets’ and ‘using this representation to derive formulas for union, intersection and complement of type-2 fuzzy sets without having to use the Extension Principle’. By using interval type-2 fuzzy sets, characterised by secondary membership functions taking values of either 0 or 1, the type reduction necessary for defuzzification of type-2 fuzzy sets is simplified [4]. Although interval type-2 fuzzy logic controllers are a lot less computationally intensive than the general type-2 FLC, there remains more computational over-

head than that required for a type-1 fuzzy logic controller, related to the number of rules fired [2],[4].

Hagras [5] implemented an interval type-2 fuzzy logic controller (FLC) and demonstrated that it outperformed a type-1 FLC. Coupland has shown that the use of geometric methods can resolve the computational overhead required in general type-2 fuzzy logic, and so allow it to be applied to time critical control problems [6]. This was demonstrated in [7], where a general type-2 FCL outperformed both an interval type-2 and a type-1 FLC, all executing the same tasks. Studies comparing type-2 and type-1 FLC performance have shown that the best results are given by the type-2 controllers [8], [9]. Hagras recently described a method to develop a type-2 FLC through embedded type-1 FLCs demonstrating that the type-2 FLC outperforms the type-1 FLCs that it was based on [10].

In previous work we investigated the properties of FLCs using four type-1 and four interval type-2 seven term controllers [11]. In this paper, we extend this work by investigating and comparing the differences between a seven, a five and two three term controllers of type-1 and interval type-2. Included in the investigation was an independent term PID controller as a reference. The micro robots used were severely constrained by low on-board memory and a 16 bit processor. Our type-2 FLS requires two type-1 calculations to be made for each membership function accessed. So significant improvements in processing load can be made by using five term and three term controllers over a seven term controller. In this work the controllers have not been optimised, in order to demonstrate the advantage of using FLCs over a PID controller.

A DC motor model suitable for micro robots was used to provide the process for the controllers. Section II describes the setup and process for the type-1 and interval type-2 FLCs, together with a description of an independent PID controller and the DC Motor model used throughout the investigation. Section II-E describes the method used to run the FLS and generate the results. Section III presents the results and Section IV is a discussion of them. Finally Section V presents conclusions and future work.

## II. SETUP AND METHOD

Nine controllers were used in the simulation. A PID controller, four type-1 controllers (three triangular MFs and a trapezoidal MF) and four type-2 controllers (three triangular MFs and a trapezoidal MF). A generic test harness was set up to run the simulation. This consisted of the controller under test and a DC motor model simulator in a closed loop

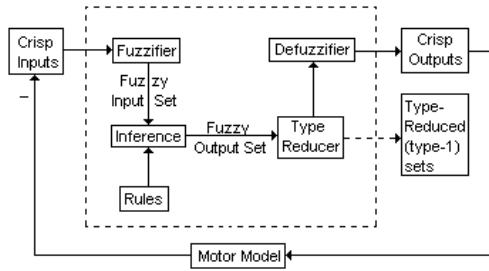


Fig. 1. Type-2 FLC Schema

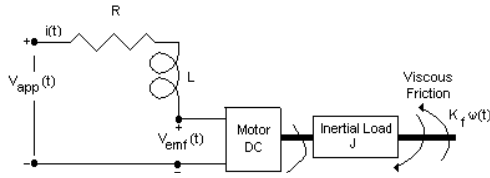


Fig. 2. DC-Motor Model

configuration. The schematic of the harness is given in Fig. 1 in the type-2 configuration. A schematic of the DC motor model is given in Fig. 2. The DC motor model was matched to the micro-robot with the following state-space values.  $R = 0.2\Omega$ ,  $L = 0.5$  H,  $K_m = 0.015$  Nm/A,  $K_b = 0.015$  VSecs/Rad,  $K_f = 0.2$  Nms,  $J = 0.01$  Kg $m^2$ .

#### A. Independent PID Controller

A PID controller was used to control the DC motor model and to provide a reference. The PID controller used in the simulation takes the independent form without integral wind-up to avoid saturation:

$$u(t) = K_p e(t) + K_i \int_0^T e(t) dt + K_d \frac{de}{dt}, \quad (1)$$

where  $u$  is the control signal or process output and  $e$  is the process error. The controller parameters are the proportional gain  $K_p$ , integral gain  $K_i$  and derivative gain  $K_d$ . The range of the controller was  $-2$  to  $2$  V, the same as the DC motor model input voltage. The process output of the model was fed back into the PID controller as the process variable and subtracted from the set point to generate the error term. In a series of benchmark experiments to determine the control constants, the PID controller was used to control the motor model. The following values were found to give the best control response,  $K_p = 100$ ,  $K_i = 200$  and  $K_d = 10$ .

#### B. Fuzzy Logic Controllers

The following generic assumptions were made in the simulation for the Mamdani controllers:

- 1) Double-input/single-output process is a non-linear input/output relationship.
- 2) The control goal is to obtain an output variable at steady state by altering the input variable. This may be noisy.

TABLE I  
SEVEN TERM FUZZY LOGIC CONTROLLER

Control Change	Error Change						
	nb	nm	ns	zr	ps	pm	pb
Error	nb	nb	nb	nb	nm	ns	zr
	nm	nb	nb	nm	ns	zr	ps
	ns	nb	nb	nm	ns	zr	ps
	zr	nb	nm	ns	zr	ps	pm
	ps	nm	ns	zr	ps	pm	pb
	pm	ns	zr	ps	pm	pb	pb
	pb	zr	ps	pm	pb	pb	pb

TABLE II  
FIVE TERM FUZZY LOGIC CONTROLLER

Control Change	Error Change				
	nb	nms	zr	pms	pb
Error	nb	nb	nb	nms	zr
	nms	nb	nb	nms	zr
	zr	nb	nms	zr	pms
	pms	nms	zr	pms	pb
	pb	zr	pms	pb	pb

- 3) A non-idealized response is expected due to the inertia of the process being controlled.

#### C. Seven Term FLC

The rule base for the seven term controller is shown in Table I, where nb - *NegBig*, nm - *NegMed*, ns - *NegSmall*, zr - *Zero*, pb - *PosBig*, pm - *PosMed* and ps - *PosSmall*.

A mix of trapezoidal and triangular membership functions were used in the seven term controller shown in Fig. 3(a). The *NegBig* and *PosBig* MFs were the trapezoids and the remainder triangles. This was due to the controller only infrequently operating at the top and bottom of the input and output ranges. The initial MFs used were determined by experiments reported previously [11]. Inputs and output used the same MFs in the controller. The method to generate the type-2 Footprint of Uncertainty (FOU) was to take the base end point values of the type-1 MFs and added an arbitrary  $\pm 10\%$  to give uncertainty intervals associated with them. For the triangles, the apex value of the type-1 MFs was used for the apex value of the type-2 MFs. This made each leg of the triangular type-2 MFs a triangle [12]. The sides of the type-2 trapezoids were parallel separated by the width of the uncertainty interval, see Fig. 3(b).

1) *Five Term FLC*: The five term controller was generated from the seven term controller, by combining the *NegMed* and *NegSmall* terms give a *NegMS* term and combining the *PosMed* and *PosSmall* terms to give a *PosMS* term. The new triangular MFs have an apex at the average of the original triangular apexes with a base of  $1.5V$  and the *Zero* term has a base of  $1V$ . The rule base for the five term controllers is given in Table II where nb - *NegBig*, nms - *NegMS*, zr - *Zero*, pms - *PosMS* and pb - *PosBig*. A mix of trapezoidal and triangular MFs were used in the controller. Inputs and output used the same MFs in the controller. The five term membership functions are given in Figures 3(c) and (d). The

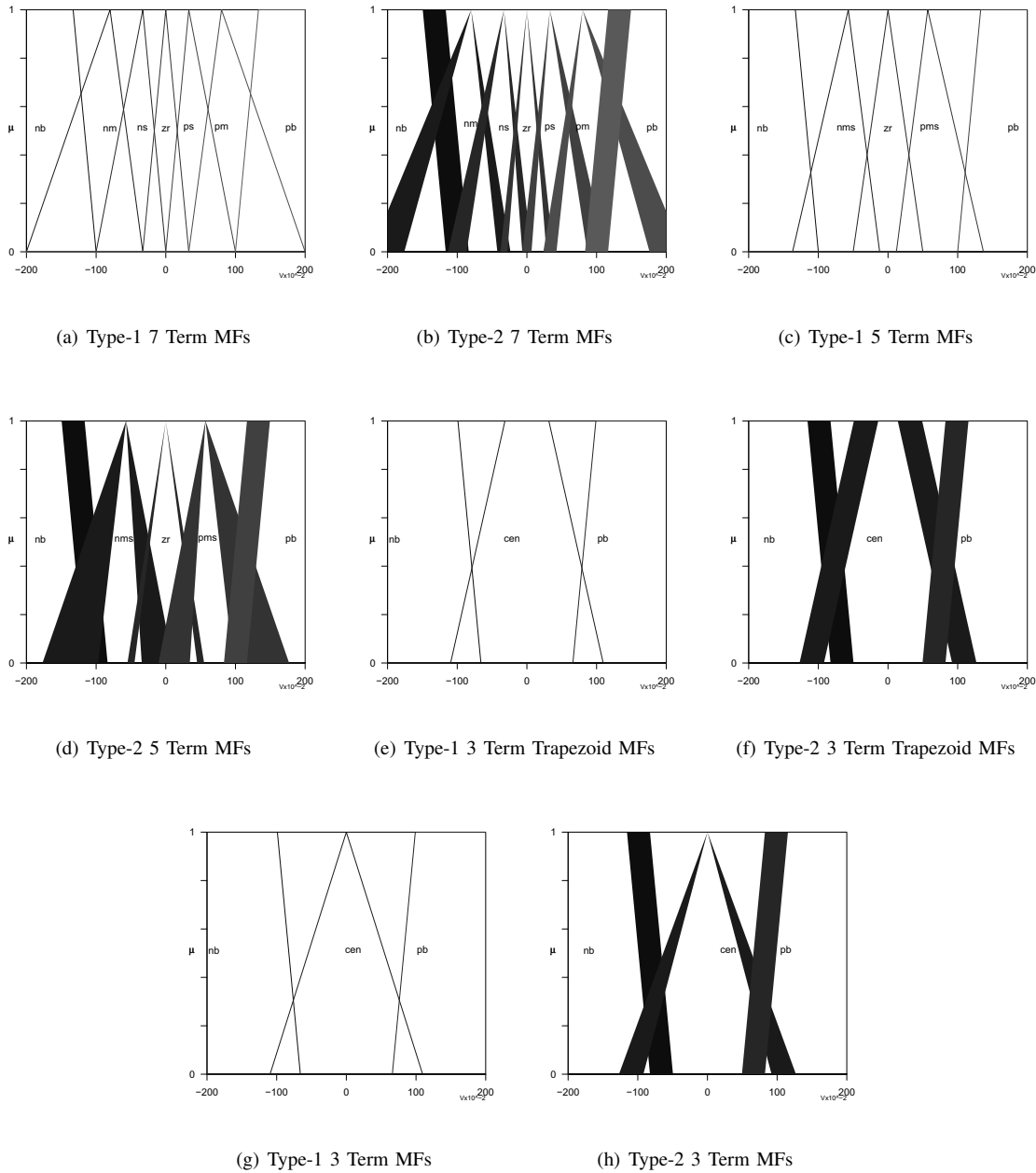


Fig. 3. Type-1 and Type-2 FLC Membership Function Shapes

TABLE III  
THREE TERM FUZZY LOGIC CONTROLLER

Control Change	Error Change		
	nb	cen	pb
Error	nb	nb	cen
	cen	nb	cen
	pb	cen	pb

type-2 MFs were generated using the 7 term method. The design of the 5 and 3 term controllers including the type-2 FOU's was arbitrary, optimisation being outside the scope of this paper.

2) *Three Term FLC*: For the three term controllers, two controllers were used. One consisting of a mix of trapezoidal and triangular MFs, see Figures 3(g) and (h), and the other

of only trapezoidal MFs shown in Figures 3(e) and (f). The trapezoidal controller was generated for comparison with the mixed controller. They were generated from the five term controller by combining the *NegMS*, *Zero* and *PosMS* terms into the *Cen* term, with the centre about zero. The rule base for the three term controllers is given in Table III, where nb - *NegBig*, cen - *Cen* and pb - *PosBig*. The uncertainty intervals were generated as +/-14% about the type-1 base end points. All the FLC type-2 surfaces are shown in Figure 4.

*D. FLC Inference and Defuzzification*

The FLCs were all set up to use the MIN rule connection method, the MIN inference action method and the MAX aggregation method. For defuzzification, the type-1 FLC used a centre of gravity method to generate a crisp value. The interval type-2 FLC used a centroid of gravity method, and

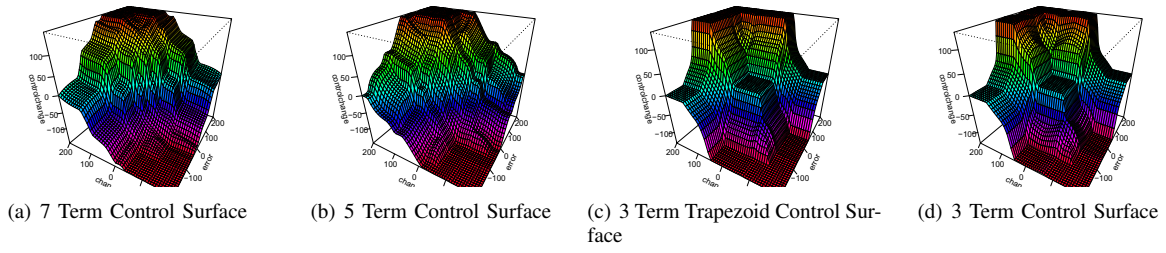


Fig. 4. Type-2 Control Surfaces

TABLE IV  
STEP RMSE FOR 7,5 AND 3 TERM CONTROLLERS

RMSE	Type-1 (Rank)	Type-2 (Rank)
Without Noise		
7Term	0.000730(1)	0.000923(2)
5Term	0.002818(8)	0.001565(7)
3Term	0.001019(6)	0.001017(5)
3TermTrap	0.001016(4)	0.001013(3)
With Noise		
7Term	0.040038(5)	0.039046(2)
5Term	0.041480(7)	0.039112(3)
3Term	0.041629(8)	0.041113(6)
3TermTrap	0.038871(1)	0.039222(4)

TABLE V  
INERTIA RMSE FOR 7,5 AND 3 TERM CONTROLLERS

RMSE	Type-1 (Rank)	Type-2 (Rank)
Without Noise		
7Term	0.000996(1)	0.001171(2)
5Term	0.002959(8)	0.001742(7)
3Term	0.001359(6)	0.001357(4)
3TermTrap	0.001357(4)	0.001354(3)
With Noise		
7Term	0.039015(1)	0.039353(3)
5Term	0.039153(2)	0.040627(7)
3Term	0.040225(5)	0.039898(4)
3TermTrap	0.040274(6)	0.041277(8)

a crisp value was obtained by averaging the upper and lower interval values.

### E. Method

The FLC and PID controllers were run for 600 timesteps of 1 ms with a step change from 0V to 1V at time  $t = 1$ . Then a second run was made increasing the inertia parameter  $J$  of the DC motor model by a factor of 10 at  $t = 300$ . This represents a load on the wheels, such as occurs when the robot hits an obstacle. Gaussian noise  $N(0,0.04)$ , which gives 1% of the motor voltage  $V_{app} \in [-2,2]$ , was added to  $V_{app}$  to generate process noise. The runs were repeated 50 times to obtain an average measurement. Execution time of the FLC is not considered since the target robot will be accessing the control surfaces via a data table lookup routine.

## III. RESULTS

The Root Mean Square Error (RMSE) between the Motor Demand and the Process Variable of the DC motor was examined. The mean values of the 7, 5 and 3 term controllers without noise and with noise are reported.

### A. FLC Step Change RMSE Results

Overall the best performing controller in the without noise set was the type-1 seven term. However, the other type-2 controllers out performed their equivalent type-1 controllers. The results for the three term controllers were all very close to each other. The five term controllers were the worst performing. In the with noise set the best controller was the type-1 three term trapezoidal controller. Otherwise the type-2 controllers out performed their equivalent type-1 controllers. The results are given in Table IV.

### B. FLC Inertia Change RMSE Results

In the without noise set the best performing controller was the seven term type-1 controller. The other type-2 controllers out performed their type-1 equivalents, as was seen in the step change without noise results. The RMSE of the three term controllers differed only by  $4 \times 10^{-6}$ . In the with noise set the type-1 seven term controller was the best. The type-1 five term controller showed the best improvement over its step change result. Contrary to expectation the type-1 controllers were better than their type-2 equivalents, except for the type-1 three term trapezoidal triangular controller. The results are given in Table V.

### C. FLC Statistical Analysis

The means and standard deviations for the type-1 and type-2 RMSEs for the step and inertia responses are given in Table VI. In both the step response cases the type-2 controllers had a lower overall RMSE. In the inertia response case without noise the type-2 controllers had a lower overall RMSE. However with noise the type-1 controllers had the lower overall RMSE. A paired t test were performed to compare the type-1 and type-2 controllers for a step change without noise. No statistical difference was found at the 95% significance level. This was repeated for the other three case with the same result.

### D. PID Controller Step and Inertia Results

The RMSE of the PID controller in the case of the step change is 44 times greater than the mean of the RMSE for all the type-1 controllers without noise and 55 times greater than the mean of the RMSE for all the type-2 controllers without noise, see Table VI. The ratios of PID to type-1 and for PID to type-2 for the mean of all the RMSE FLCs with noise are 1.86 and 1.89 respectively. For the inertia change case the RMSE of the PID controller is 16 times greater than the mean of the RMSE for all the type-1 controllers without noise and 19 times greater than the mean of the RMSE for

TABLE VI  
MEAN AND SD OF ALL FLCs

Step Response	Without Noise		With Noise	
	Mean	SD	Mean	SD
PID	0.062436	-	0.075369	-
Mean Type-1	0.001396	0.000958	0.040504	0.001304
Mean Type-2	0.001130	0.000293	0.039713	0.000941
Inertia Response	Without Noise		With Noise	
	Mean	SD	Mean	SD
PID	0.207588	-	0.214935	-
Mean Type-1	0.001667	0.000878	0.039667	0.000675
Mean Type-2	0.001406	0.000241	0.040289	0.000840

TABLE VII  
KEY TO TYPE-1 AND TYPE-2 FLC RESPONSE GRAPHS

Key	Series Without Noise
T2_7	Type-2 Seven Term Trapezoidal Triangular MF
T2_5	Type-2 Five Term Trapezoidal Triangular MF
T2_trap	Type-2 Three Term Trapezoidal MF
T2_tri	Type-2 Three Term Trapezoidal Triangular MF
T1_7	Type-1 Seven Term Trapezoidal Triangular MF
T1_5	Type-1 Five Term Trapezoidal Triangular MF
T1_trap	Type-1 Three Term Trapezoidal MF
T1_tri	Type-1 Three Term Trapezoidal Triangular MF

all the type-2 controllers without noise. The ratios of PID to type-1 and for PID to type-2 for the mean of all the RMSE FLCs with noise are 5.8 and 5.3 respectively.

#### E. Comparison of Type-2 Controllers with Type-1 and PID

In Figure 5(a) the without noise responses for all eight FLCs to a step change are plotted over 120 timesteps. The two type-1 and the two type-2 three term FCL step change responses are superimposed due to the closeness of the results. In Figure 5(b) the without noise responses for all eight FLCs to an inertia change are plotted over 120 timesteps. The two type-1 and the two type-2 three term plus the five term type-1 FLC inertia change responses are superimposed.

The keys used to denote the controller responses in Figure 5 are given in Table VII.

In response to the step change the lowest overshoot was given by the three term type-2 trapezoidal controller — 0.11% at  $t = 1$ . This controller crossed back over the setpoint (SP) at  $t = 7$ , and reached a minimum at  $t = 40$ . It had reached a steady state by  $t = 158$ . The best time to steady state was from the seven term type-1 controller — 99.98% in 104 timesteps. When responding to the inertia change all the controllers dipped to 99.616% of the SP. The seven term type-1 crossed back at  $t = 357$  overshooting by 0.006% of the SP. It returned to the setpoint at  $t = 445$  and reached a steady state of 99.97% of SP. The type-2 controller crossed back at  $t = 383$  overshooting by 0.0002% of SP, and returned to the setpoint at  $t = 461$ , reaching a steady state of 99.96% of SP. Both types of five and three term controllers behave in an overdamped way having maximum values of 99.89% of SP at  $t = 600$ . The PID controller reached 90% of SP at  $t = 9$ , and crossed the SP at  $t = 37$ . It reached a maximum

of 100.079% of SP at  $t = 54$ . The PID controller reached a steady state of 100.0016% of SP at  $t = 139$ . When the inertia change occurred the controller went to 0% of SP at  $t = 301$  and reached 90% of SP at  $t = 331$ . It crossed the SP at  $t = 334$  and reached a maximum of 100.479% at  $t = 367$ . It had not reached a steady state by  $t = 600$ .

In Figures 5(c) and (d), the noisy response to the inertia change of the controllers is given. When the inertia change occurred no observable difference was detected in the controllers response.

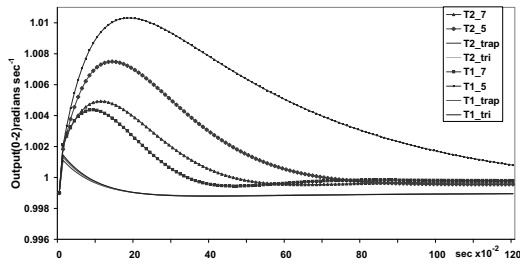
#### IV. DISCUSSION

It can be seen in Tables IV and V that the overall performance of the two five term controllers were the worst. The intersection of their membership functions was much lower than the MFs of the seven term controllers. This suggests that in future experiments the intersections for these controllers needs to be higher. The improvement of the type-2 five term controller with noise over its without noise equivalent is noted, especially when it is not repeated for the inertia change results. This is currently not understood. Generally like for like, the three term controllers RMSEs were between the RMSEs of the seven term and five term controllers.

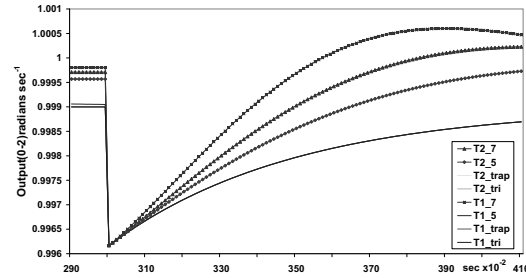
When noise was applied, as expected the type-2 controllers were usually better than the type-1 controllers for the step change response. Surprisingly in the inertia change response with noise, the type-1 controllers were generally better. A possible cause for this is that the level of noise is too low. The results of increasing the load on the motor correspond with those obtained by Hagrais [10], with the type-2 FLC's being more damped than the type-1 controllers. When noise was applied the overall means of the type-1 and type-2 FLC RMSEs increased by over twenty five times, where as the PID RMSE increased by 21% in the step response and by 3.5% for the inertia response. For the type-2 controllers with triangular MFs, the created FOU decreases towards the peak of the triangles reducing the uncertainty to zero. This was kept so as to be consistent with the original seven term controllers. However, for the three term type-2 controller with the triangular cen MF, there is less uncertainty around zero. Statistically there was no significant difference between the type-1 and type-2 controllers at the 95% level for both the step change and the inertia change. This suggests that either type-1 or type-2 controllers can be used.

#### V. CONCLUSION AND FUTURE WORK

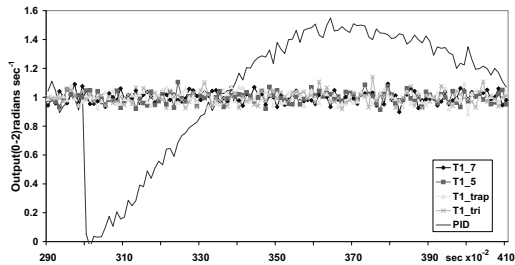
Our study shows that the type-1 and type-2 controllers cannot be statistically distinguished from each other. However, in the case of an inertia change the type-1 controllers out performed the type-2 controllers. The work carried out demonstrates that, in simulation of a micro robot DC Motor, seven and five term controllers can be nominally replaced by ideally, a three term controller of type-1 without appreciable loss of control. The three term membership functions can either be all trapezoidal or the central membership function triangular. The results show that the three term controllers are as stable as the seven and five term controllers when a



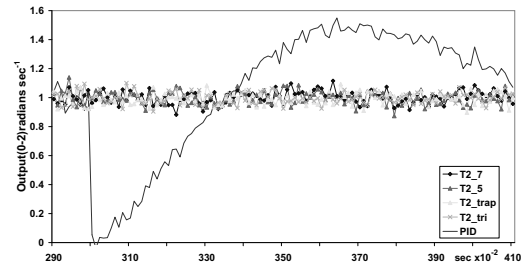
(a) The response to all eight controllers to an input step change



(b) The response to all eight controllers to a change in inertia



(c) Noisy type-1 response to a change in inertia



(d) Noisy type-2 response to a change in inertia

Fig. 5. FLC Step and Inertia Change Responses

step change or load is applied. When noise is applied the three term controllers perform equally with the seven and five term controllers. Obviously these results are specific to our motor model and not transferable to all DC motors. In future work it is intended that three term type-1 and type-2 controllers are put into micro robots and evaluated using the real world environment. It is also suggested that the five term controllers are further investigated to determine the reason for their poor performance. A possible cause of the better performance of the type-1 controllers over the type-2 controllers, in the inertia change case, is that the level of noise applied was too low. Further experiments need to be performed at higher levels of noise to determine when the type-1 controllers fail. Also changes to the membership functions to add in uncertainty at the peaks and their effect is required to be investigated.

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